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CANKAYA UNIVERSITY
FACULTY OF ENGINEERING AND ARCHITECTURE
MECHANICAL ENGINEERING DEPARTMENT

ME 211 THERMODYNAMICS I

CHAPTER : EXAMPLES SOLUTIONS

1. Heat is transferred to a heat engine from a furnace at a rate of 80 MW. If the rate of waste heat rejection to a nearby river is 50 MW, determine the net power output and the thermal efficiency for this heat engine.

Assumptions: Heat losses through pipes and other components are negligible.

$$\dot{Q}_H = 80 \text{ MW} \text{ and } \dot{Q}_C = 50 \text{ MW}$$

The net power output of the heat engine is:

$$\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C = (80 - 50) \text{ MW} = 30 \text{ MW}$$

The thermal efficiency is:

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = \frac{30 \text{ MW}}{80 \text{ MW}} = 0.375 \text{ or } 37.5\%$$

Note that the heat engine converts 37.5 percent of the heat it receives to work.

2. A household refrigerator with $\beta = 1.5$, removes heat from the refrigerated space at a rate of 60 kJ/min. Determine:

- (a) the electric power consumed by the refrigerator and
- (b) the rate of heat transfer to the kitchen air.

The β and the refrigeration rate of a refrigerator are given. The power consumption and the rate of heat rejection are to be determined.

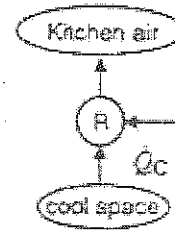
Assumptions The refrigerator operates steadily.

Analysis (a) Using the definition of the coefficient of performance, the power input to the refrigerator is determined to be

$$\dot{W}_{\text{cycle}} = \frac{\dot{Q}_C}{\beta} = \frac{60 \text{ kJ/min}}{1.5} = 40 \text{ kJ/min} = 0.67 \text{ kW}$$

(b) The heat transfer rate to the kitchen air is determined from the energy balance,

$$\dot{Q}_H = \dot{Q}_C + \dot{W}_{\text{cycle}} = 60 + 40 = 100 \text{ kJ/min}$$



3. Determine the γ of heat pump that supplies energy to a house at a rate of 8000 kJ/h for each kW of electric power it draws. Also, determine the rate of energy absorption from the outdoor air.

The rate of heat supply of a heat pump per kW of power it consumes is given. The γ and the rate of heat absorption from the cold environment are to be determined.

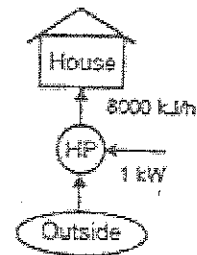
Assumptions The heat pump operates steadily.

Analysis The coefficient of performance of the refrigerator is determined from its definition,

$$\gamma = \frac{\dot{Q}_H}{\dot{W}_{\text{cycle}}} = \frac{8000 \text{ kJ/h}}{1 \text{ kW}} \left(\frac{1 \text{ kW}}{3600 \text{ kJ/h}} \right) = 2.22$$

The rate of heat absorption from the surrounding air, per kW of power consumed, is determined from the energy balance,

$$\dot{Q}_C = \dot{Q}_H - \dot{W}_{\text{cycle}} = (8,000 \text{ kJ/h}) - (1)(3600 \text{ kJ/h}) = 4400 \text{ kJ/h}$$



4. A Carnot heat engine receives 650 kJ of heat from a source of unknown temperature and rejects 200 kJ of it to a sink at 17 °C. Determine:

- (a) the temperature of the source and
- (b) the thermal efficiency of the heat engine.

The sink temperature of a Carnot heat engine and the rates of heat supply and heat rejection are given. The source temperature and the thermal efficiency of the engine are to be determined.

Assumptions: The Carnot heat engine operates steadily.

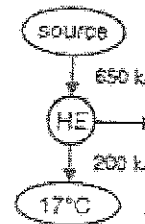
Analysis: (a) For reversible cyclic devices we have $\left(\frac{Q_H}{Q_C}\right)_{rev} = \left(\frac{T_H}{T_C}\right)$

Thus the temperature of the source T_H must be

$$T_H = \left(\frac{Q_H}{Q_C}\right)_{rev} T_C = \left(\frac{650 \text{ kJ}}{200 \text{ kJ}}\right)(290 \text{ K}) = 942.5 \text{ K}$$

(b) The thermal efficiency of a Carnot heat engine depends on the source and the sink temperatures only, and is determined from

$$\eta_{max} = 1 - \frac{T_C}{T_H} = 1 - \frac{290 \text{ K}}{942.5 \text{ K}} = 0.69 \text{ or } 69\%$$



5. An experimentalist claims that, based on his measurements, a heat engine receives 300 Btu of heat from a source of 900 R, converts 160 Btu of it to work, and rejects the rest as waste heat to sink at 540 R. Are these measurements reasonable? Why?

An inventor claims to have developed a heat engine. The inventor reports temperature, heat transfer, and work output measurements. The claim is to be evaluated.

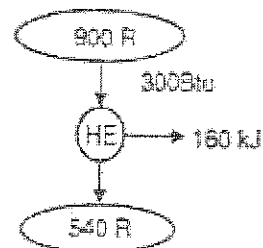
Analysis The highest thermal efficiency a heat engine operating between two specified temperature limits can have is the Carnot efficiency, which is determined from

$$\eta_{\max} = 1 - \frac{T_C}{T_H} = 1 - \frac{540 \text{ R}}{900 \text{ R}} = 0.40 \quad \text{or} \quad 40\%$$

The actual thermal efficiency of the heat engine in question is

$$\eta = \frac{W_{\text{cycle}}}{Q_H} = \frac{160 \text{ Btu}}{300 \text{ Btu}} = 0.533 \quad \text{or} \quad 53.3\%$$

which is greater than the maximum possible thermal efficiency. Therefore the claim is false.



6. An inventor claims to have developed a heat engine that receives 800 kJ of heat from a source at 400K and produces 250 kJ of net work while rejecting the waste heat to a sink at 300K. Is this reasonable?

To evaluate these claims, we check to make sure that neither the First Law nor Second Law has been violated.

$$\text{First Law: } Q_H - Q_C = W_{\text{cycle}}$$

$$Q_C = Q_H - W_{\text{cycle}} = 800 \text{ kJ} - 250 \text{ kJ} = 550 \text{ kJ}$$

$$\eta = \frac{W_{\text{cycle}}}{Q_H} = 0.312 = 31.2\%$$

This at least sounds reasonable. If the efficiency were greater than one, we would know right away that the claim was a hoax.

Let's see if 31.2% is reasonable for an engine operating between the given temperature reservoirs.

$$T_H = 673 \text{ K } T_C = 573 \text{ K}$$

$$\eta_{\text{max}} = 1 - \frac{T_C}{T_H} = 1 - \frac{573}{673} = 0.149 = 14.9\%$$

The inventor's claim is false. The maximum efficiency possible for an engine operating between these two reservoirs is only 14.9%.

7. A heat pump is used to heat a house and maintain it at 20°C . When the outdoor air temperature is -5°C , the house loses heat at a rate of 75000 kJ/hr . Find the minimum power required to operate the heat pump.

The minimum power required will be fulfilled by a reversible heat pump. That is one with maximum efficiency. For a reversible heat pump, γ is a function of the temperature reservoirs only:

$$T_H = 293 \text{ K} \quad T_C = 268 \text{ K}$$

$$\gamma_{\max} = \frac{1}{1 - T_C/T_H} = \frac{1}{1 - 268/293} = 11.72$$

$$\dot{W}_{\text{cycle}} = \frac{75,000 \text{ kJ/hr}}{\gamma_{\max}} = 1.778 \text{ kW}$$

8. A heat pump with a γ of 3.2 consumes 5 kW of power. When the heat pump is turned on, the temperature inside the house is 7°C . If the mass of the house's contents is equivalent to a mass of 1500 kg of air ($c_v = 0.72 \text{ kJ/kg}\cdot^\circ\text{C}$, $c_p = 1.0 \text{ kJ/kg}\cdot^\circ\text{C}$), how long will it take to raise the temperature of the house to 22°C ?

$$\gamma = 3.2$$

$$\dot{W}_{\text{cycle}} = 5 \text{ kW}$$

$$\dot{Q}_H = \dot{W}_{\text{cycle}} \cdot \gamma = 5 \text{ kW} (3.2) = 16 \text{ kW}$$

This is power – energy/time.

Now we need to find how much energy is required to heat the house.

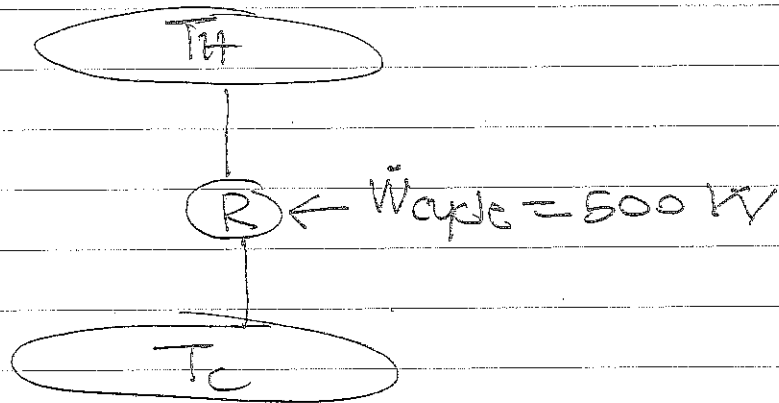
$$Q_{\text{required}} = m C_v (T_2 - T_1) = 1500 \text{ kg} (0.72 \text{ kJ/kg}\cdot^\circ\text{C}) (22 - 7)$$

$$Q_{\text{required}} = 16,200 \text{ kJ}$$

$$\text{Time} = \frac{Q_{\text{required}}}{\dot{Q}_H} = \frac{16,200 \text{ kJ}}{16 \text{ kW}} = 16.875 \text{ minutes}$$

9. A Carnot refrigerator operates in a room in which the temperature is 25°C . The refrigerator consumes 500 W of power when operating and has β of 4.5 . Determine:

- (a) the rate of heat removal from the refrigerated space and
 (b) the temperature of the refrigerated space.



$$\begin{aligned}\dot{Q}_c &= \beta_{\max} \dot{W}_{\text{cycle}} = (4.5)(0.5\text{ kW}) = 2.25\text{ kW} \\ &= 2.25 \frac{\text{kJ}}{\text{s}} \cdot \frac{60\text{ s}}{\text{min}} \\ &= 135 \text{ kJ/min}\end{aligned}$$

$$\text{b) } \beta_{\max} = \beta_{\text{rev}} = \frac{1}{\frac{T_H}{T_C} - 1} \Rightarrow$$

$$4.5 = \frac{1}{\frac{298}{T_C} - 1} \Rightarrow T_c = 243.81\text{ K}$$

$$T_c = -29.3^\circ\text{C}$$

Heat removal by refrigerator

$$\begin{aligned}\dot{Q}_c &= \beta \dot{W}_{\text{cycle}} = (8.37)(595.2 \frac{\text{kJ}}{\text{min}}) \\ &= 4982 \text{ kJ/min}\end{aligned}$$

b) Total heat rejection

Heat rejected by HE

$$\dot{Q}_H = \dot{W}_{\text{cycle}} + \dot{Q}_c$$

$$\dot{Q}_c = 800 - 595.2 = 204.8 \text{ kJ/min}$$

Heat rejected by refrigerator

$$\dot{W}_{\text{cycle}} + (\dot{Q}_c)_R = (\dot{Q}_H)_R$$

$$(\dot{Q}_H)_R = 4982 + 595.2 = 5577.2 \text{ kJ/min}$$

$$\dot{Q}_{\text{TOTAL}} = (\dot{Q}_H)_R + (\dot{Q}_H)_{\text{HE}}$$

$$= 5577.2 + 204.8$$

$$= 5782 \text{ kJ/min}$$

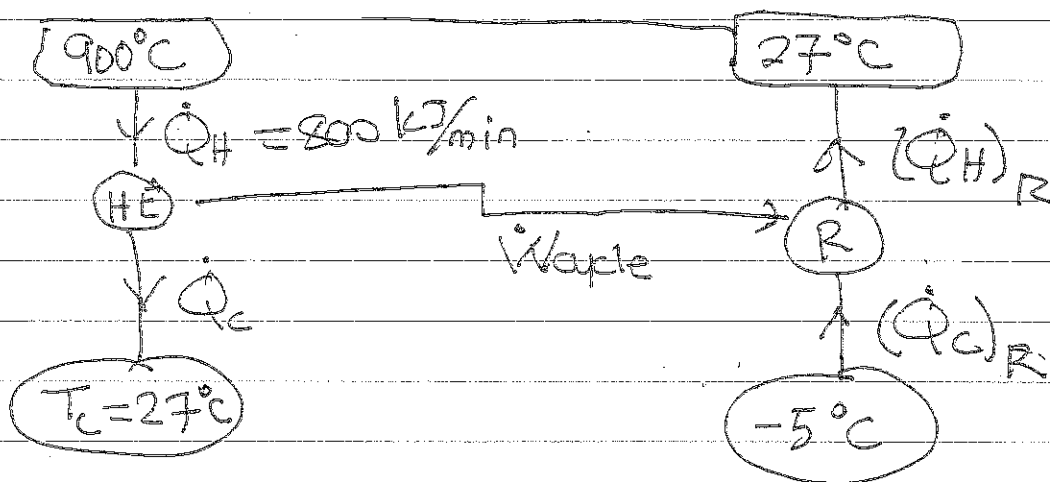
10. A Carnot heat engine receives heat from a reservoir at 900°C at a rate of 800 kJ/min and rejects the waste heat to the ambient air at 27°C . The entire work output of the heat engine is used to drive a refrigerator that removes heat from the refrigerated space at -5°C and transfers it to the same ambient air at 27°C . Determine:

- the maximum rate of heat removal from the refrigerated space and
- the total rate of heat rejection to the ambient air.

$$a) \quad \eta_{\text{rev}} = \eta_{\text{max}} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{1173} = 0.744$$

74.4%

$$\dot{W}_{\text{max}} = \eta_{\text{max}} \dot{Q}_H = (0.744)(800\text{ kJ/min}) = 595.2 \frac{\text{kJ}}{\text{min}}$$



Rate of heat removal from refrigerated space is maximum if Refrigerator is reversible.

$$\beta_{\text{max}} = \frac{1}{\frac{T_H}{T_C} - 1} = \frac{1}{\frac{300}{268} - 1} = 837$$