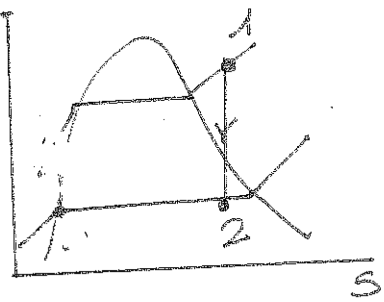
CANKAYA UNIVERSITY
FACULTY OF ENGINEERING
MECHANICAL ENGINEERING DEPARTMENT

ME 211 THERMODYNAMICS I

CHAPTER 5
EXAMPLE SOLUTIONS

1) A rigid tank contains 5 kg of Refrigerant-134a initially at 20°C and 140 kPa. The refrigerant is now cooled while being stirred until its pressure drops to 100 kPa. Determine the entropy change of the refrigerant during this process.



State 1:

$$\left.\begin{array}{l} P_1 = 140\,\text{kPa} \\ T_1 = 20\,^\circ\text{C} \end{array}\right\} s_1 = 1.0532\,\text{kJ/kg.K},\ v_1 = 0.1652\,\text{m}^3/\text{kg}$$

State 2:

$$\left.\begin{array}{l} P_2 = 100\,\text{kPa} \\ v_2 = v_1 \end{array}\right\} v_f = 0.0007258\,\text{m}^3/\text{kg},\ v_g = 0.1917\,\text{m}^3/\text{kg}$$

The refrigerant is a saturated liquid-vapor mixture at the final state since $v_f < v_2 < v_g$ at 100 kPa pressure. Therefore, we need to determine the quality first:

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.1652 - 0.0007258}{0.1916 - 0.0007258} = 0.861$$

Thus,

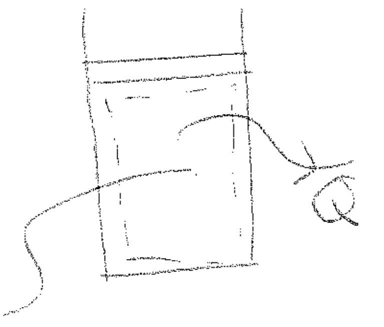
$$s_2 = s_f + x_2 s_{fg} = 0.0678 + (0.861)(0.9395 - 0.0678) = 0.8183\,\text{kJ/kg.K}$$

Then the entropy change of the refrigerant during this process is:

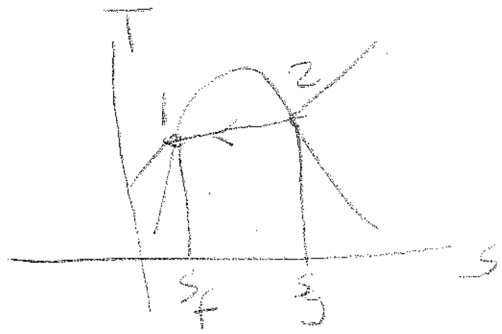
$$\Delta S = m(s_2 - s_1) = (5\text{kg})(0.8183 - 1.0532)\,\text{kJ/kg.K} = -1.175\,\text{kJ/K}$$

2) Suppose that 1 kg of saturated water vapor at 100 ^{0}C is condensed to saturated liquid at 100 ^{0}C in a constant pressure process by heat transfer to the surrounding air, which is at 25 ^{0}C. What is the net increase in entropy of water plus surroundings?

Solution



$T_{sur} = 25°C$



H_2O
$m = 1kg$
$T = 100°C$

$$Q - W = U_2 - U_1$$

$$Q = (U_2 - U_1) + p(V_2 - V_1) = H_2 - H_1 = m(h_2 - h_1)$$

$$= m h_{fg} = (1)(2257) = 2257 \text{ kJ}$$

$$\Delta S_{H_2O} = -\frac{Q}{T} = \frac{2257}{373.15} = -6.0485 \text{ kJ/K}$$

$$\Delta S_{sur} = \frac{Q}{T_0} = \frac{2257}{298.15} = 7.57 \text{ kJ/K}$$

$$\Delta S_{net} = \Delta S_{H_2O} + \Delta S_{sur} = 7.57 - 6.0485$$

$$= 1.5215 \text{ kJ/K}$$

Example: 3

Oxygen is heated from 300 K to 1500 K. Assume that during this process the pressure dropped from 200 kPa to 150 kPa. Calculate the change in entropy per kilogram.

Answer:

From Table A-23:

$\bar{s}^{\circ}(300\,K) = 205.213\,kJ/kmol\cdot K$

$\bar{s}^{\circ}(1500\,K) = 257.965\,kJ/kmol\cdot K$

and

$$s^{\circ} = \bar{s}^{\circ}/M \Rightarrow s^{\circ}(300\,K) = \frac{205.213\,kJ/kmol}{32\,kg/kmol} = 6.413\,kJ/kg$$

$$s^{\circ} = \bar{s}^{\circ}/M \Rightarrow s^{\circ}(1500\,K) = \frac{257.965\,kJ/kmol}{32\,kg/kmol} = 8.061\,kJ/kg$$

Using Eq. for ideal gas

$$s(T_2, p_2) - s(T_1, p_1) = s^{\circ}(T_2) - s^{\circ}(T_1) - R\ln\frac{p_2}{p_1}$$

with

$$R = \frac{\bar{R}}{M} = \frac{8.314\,kJ/kmol\cdot K}{32\,kg/kmol} = 0.2598\,kJ/kg\cdot K$$

so

$$s_2 - s_1 = (8.061 - 6.413) - 0.2598\,ln\left(\frac{150}{200}\right)$$

$$s_2 - s_1 = 1.723\,kJ/kg\cdot K$$

Method 2

Assume constant C_p

Evaluate C_p at average temperature

$\bar{T} = \frac{1}{2}(T_1+T_2) = \frac{1}{2}(1500+300) = 900\,K$

At $\bar{T} = 900\,K$ $\quad c_p = 1.074\ kJ/kg\,K$

R for O_2 : $R = \frac{\bar{R}}{M}$

$$= \frac{8.314\ kJ/kmol\,K}{32\ \frac{kg}{kmol}}$$

$$= 0.2598\ \frac{kJ}{kg\,K}$$

$$s_2 - s_1 = c_p \ln(T_2/T_1) - R \ln(P_2/P_1)$$

$$= (1.074) \ln\left(\frac{1500}{300}\right) - 0.2598 \ln\left(\frac{150}{200}\right)$$

$$= 1.684\ \frac{kJ}{kg\,K}$$

Example: 4

Calculate the change in entropy per kilogram as air heated from 300 K to 600 K while pressure drops from 400 kPa to 300 kPa.

Assuming

(a) constant specific heat
(b) variable specific heat

Answer: $\bar{T} = \frac{T_1+T_2}{2} = 450K$ $\quad \bar{C}_p = 1.020\ kJ/kgK$

(a) At 300 K
$c_p = 1.005\,kJ/kg.K$

$$s_2 - s_1 = c_p\,ln\frac{T_2}{T_1} - R\,ln\left(\frac{p_2}{p_1}\right) = \overset{1.02}{\cancel{1.005}}\,ln\left(\frac{600}{300}\right) - 0.287\,ln\left(\frac{300}{400}\right)$$

$$s_2 - s_1 = 0.7792\,kJ/kg\cdot K$$

(b) From Table A-22: Use table for variable specific heat

$s_1^o = 1.70203\,kJ/kg\cdot K$ at 300 K

$s_2^o = 2.40902\,kJ/kg\cdot K$ at 600 K

$$s_2 - s_1 = \left(s_2^o - s_1^o\right) - R\,ln\left(\frac{p_2}{p_1}\right) = (2.40902 - 1.70203) - 0.287\,ln\left(\frac{300}{400}\right)$$

$$s_2 - s_1 = 0.7896\,kJ/kg\cdot K$$

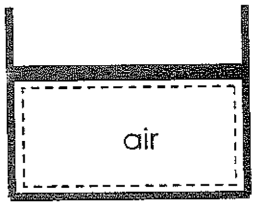
Example: 5

One kilogram of air is contained in a cylinder fitted with a piston at a pressure of 400 kPa and a temperature of 600 K. The air is expanded to 150 kPa in a reversible adiabatic process. Calculate the work done by air.

Answer:

Given Data:

$P_1 = 400\,\text{kPa} = 0.4\,\text{MPa}$, $T_1 = 600\,\text{K}$, $P_2 = 150\,\text{kPa} = 0.15\,\text{MPa}$



For closed system:

$$\underbrace{Q}_{=0} - W = m(u_2 - u_1) \qquad \text{First Law}$$

$$s_2 = s_1 \qquad \text{Second Law}$$

State 1: $u_1 = 434.78\,\text{kJ/kg}$

$$s_1^o = 2.40902\,\text{kJ/kg}\cdot\text{K}$$

$$s_2 - s_1 = 0 = \left(s_2^o - s_1^o\right) - R\,ln\left(\frac{p_2}{p_1}\right)$$

$$0 = \left(s_2^o - 2.40902\right) - 0.287\,ln\left(\frac{150}{400}\right)$$

$$s_2^o = 2.12752\,\text{kJ/kg}\cdot\text{K}$$

From Table A-22 for $s_2^o = 2.12752\,\text{kJ/kg}\cdot\text{K}$

$$T_2 = 457.1\,\text{K} \text{ and } u_2 = 327.84\,\text{kJ/kg}$$

Therefore,

$$-W = -\frac{W}{m} = u_2 - u_1 = 327.84\,\text{kJ/kg} - 434.78\,\text{kJ/kg}$$

$$W = 106.94\,\text{kJ/kg}$$

Method 2

Assume constant specific heats

$m = 1\,kg$

$P_1 = 400\,kPa$ $\qquad$ $P_2 = 150\,kPa$

$T_1 = 600\,K$ $\qquad$ $s_2 = s_1$

$\underset{0}{\cancel{Q}} - W = m(u_2 - u_1)$

$-W = mc_v(T_2 - T_1)$

Reversible adiabatic $\Rightarrow$ isentropic : $s_1 = s_2$

$\underset{0}{\cancel{s_2 - s_1}} = c_p \ln(T_2/T_1) - R\ln(P_2/P_1)$

so $T_2 = T_1 (P_2/P_1)^{\frac{k-1}{k}}$

$k \cong 1.4$

for air

$T_2 = 600(150/400)^{0.285} = 453.6\,K$

Estimate c_p at $500\,K$

$c_v \approx 0.742\,kJ/kg\,K$

$-W = mc_v(T_2 - T_1)$

$-\frac{W}{m} = c_v(T_2 - T_1) = (0.742)(453.6 - 600)$

$(W/m) = 108.7\,kJ/kg$

Example 6

Calculate Δs for air modeled as an ideal gas going from $T_1 = 300$ K and $p_1 = 1$ bar, to $T_2 = 1000$ K and $p_2 = 3$ bar.

From Table A-22 (Ideal Gas Properties of Air)

$$s^\circ(300\,\mathrm{K}) = 1.70203\,\mathrm{kJ/kg.K}$$

$$s^\circ(1000\,\mathrm{K}) = 2.96770\,\mathrm{kJ/kg.K}$$

$$s_2 - s_1 = s^\circ(T_2) - s^\circ(T_1) - R\ln\left(\frac{p_2}{p_1}\right)$$

$$s_2 - s_1 = (2.96770 - 1.70203)\left(\frac{\mathrm{kJ}}{\mathrm{kg.K}}\right) - \frac{8.314}{28.97}\left(\frac{\mathrm{kJ}}{\mathrm{kg.K}}\right)\ln\left(\frac{3\,\mathrm{bar}}{1\,\mathrm{bar}}\right)$$

$$\Rightarrow s_2 - s_1 = 0.9504\,\mathrm{kJ/kg.K}$$

Method 2

Assume constant c_p

Evaluate c_p at $\bar{T} = \frac{1}{2}(1000+300) = 650\,\mathrm{K}$

$$\therefore c_p = 1.063\ \mathrm{kJ/kg\,K}$$

$$R = \frac{\bar{R}}{M} = \frac{8.314\ \mathrm{kJ/kmol\,K}}{28.97\ \frac{\mathrm{kmol}}{\mathrm{kg}}} \cong 0.287\ \frac{\mathrm{kJ}}{\mathrm{kg\,K}}$$

$$s_2 - s_1 = c_p \ln(T_2/T_1) - R\ln(P_2/P_1)$$

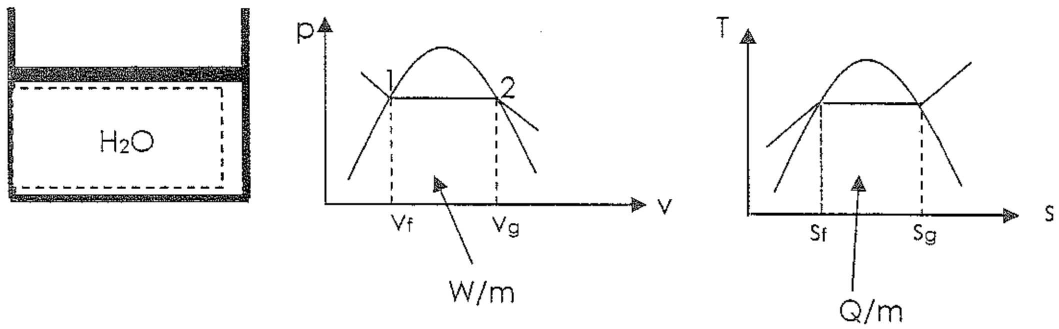
$$= (1.063)\ln(1500/300) - 0.287\ln(3/1)$$

$$= 0.9645\ \frac{\mathrm{kJ}}{\mathrm{kg\,K}}$$

Example: 7

Water initially a saturated liquid at 100 °C is contained in a piston cylinder assembly. The water undergoes a process to the corresponding saturated vapor state during which the piston moves freely in the cylinder. If the change in state is brought about by heating the water as it undergoes an internally reversible process at constant temperature and pressure, determine work and heat transfer per unit mass, each in kJ/kg.

Answer:



$$\frac{W}{m} = \int_f^g p\,dv = p(v_g - v_f) = (1.014\,\text{bar})\left(\frac{10^5\,\text{Pa}}{1\,\text{bar}}\right)\left(\frac{1\,\text{kJ}}{10^3\,\text{N.m}}\right)\left(1.673 - 1.0435\times10^{-3}\right)$$

$$\frac{W}{m} = 170\,\text{kJ/kg}$$

$$\frac{Q}{m} = \int_f^g T\,ds = T(s_g - s_f) = (373.15\,\text{K})(7.3549 - 1.3069)\,\text{kJ/kg}\cdot\text{K}$$

$$\frac{Q}{m} = 2257\,\text{kJ/kg}$$

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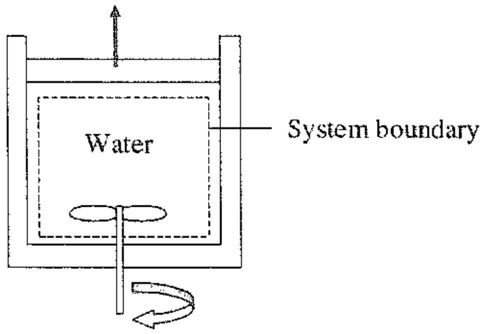
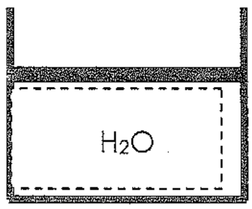
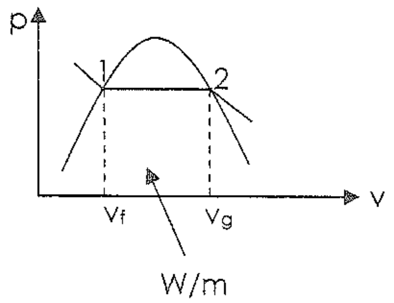
Example:

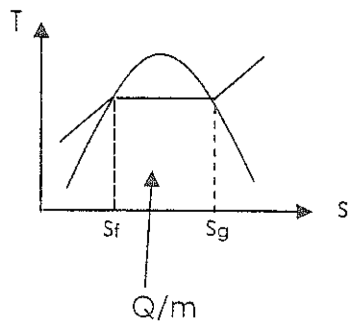
Water in a piston-cylinder assembly is stirred so that a change of state from saturated liquid at 100 °C to saturated vapor at 100 °C is effected adiabatically. What is the entropy production per unit mass, i.e., $\frac{\sigma}{m}$?



Answer:







For $Q = 0$, $\Delta KE = \Delta PE = 0$

From Eq. 6.27,

$$S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

$$\Delta s = \frac{1}{m}\int_1^2 \left(\frac{\delta Q}{T}\right)_b + \frac{\sigma}{m}$$

here $\int_1^2 \left(\frac{\delta Q}{T}\right)_b = 0$

$$\frac{\sigma}{m} = s_g - s_f \quad \text{(in Table A-2 at 100 °C)}$$

$$\frac{\sigma}{m} = (7.3549 - 1.3069)\,\text{kJ/kg}\cdot\text{K} = 6.048\,\text{kJ/kg}\cdot\text{K}$$

Example: 8

Steam enters an adiabatic turbine at 5 MPa and 450 °C and leaves at a pressure of 1.5 MPa. Determine the work output of the turbine per unit mass of steam if the process is reversible. adiabatic,

Answer:

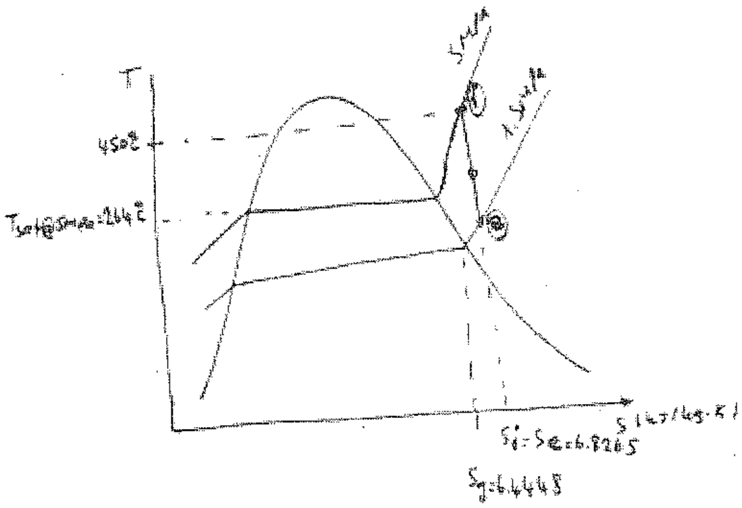
The power output of the turbine is determined from the rate form of the energy balance, Eq. 4.15,

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right) \quad \text{(kW)}$$

For steady state and one inlet one exit process, neglecting kinetic and potential energies:

$$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{m}(h_e - h_i)$$

For adiabatic process: $\dot{W}_{cv} = \dot{m}(h_i - h_e)$



State 1:

$$\left.\begin{array}{l} P_i = 5\,\text{MPa} \\ T_i = 450\,^\circ\text{C} \end{array}\right\} h_i = 3315.8\,\text{kJ/kg},\ s_i = 6.8265\,\text{kJ/kg}\cdot\text{K}$$

State 2:

$P_2 = 15$ bar $s_f = 2.3150 \frac{kJ}{kg}$

$$\left.\begin{array}{l} P_i = 1.5\,\text{MPa} \\ s_e = s_i \end{array}\right\} h_e = 2986.5\,\text{kJ/kg}$$

$s_g = 6.4448 \frac{kJ}{kg}$

Then the work output of the turbine per unit mass of the steam becomes:

$s_2 > s_g$

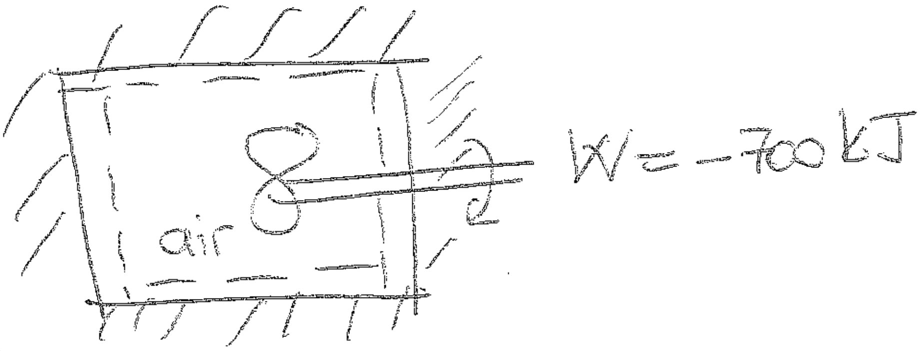
superheated

$$w_{cv} = h_i - h_e = 3315.8 - 2986.5 = 329.3\,\text{kJ/kg}$$

10) Two m^3 of air in a rigid, insulated container fitted with a paddle wheel is initially at 293 K, 200 kPa. The air receives 710 kJ by work from the paddle wheel. Assuming the ideal gas model with c_v = 0.72 kJ/kg .K, determine for the air

(a) the mass, in kg,
(b) final temperature, in K,
(c) the amount of entropy produced, in kJ/K.

Solution



1- Closed system
2- insulated, Q = 0
3- $\Delta KE = \Delta PE = 0$
4- air is modeled as ideal gas

$$m = \frac{P\forall}{RT_1} = \frac{(300 \times 10^3\ N/m^2)(2\ m^3)}{\frac{(8.314\ kJ/kmol\ K)}{(28.97\ \frac{kg}{kmol})}(293K)} = 4.76\ kg$$

$$\cancel{Q}^{0} - W = U_2 - U_1 = mc_v(T_2 - T_1)$$

$$\therefore T_2 = T_1 - \frac{W}{mc_v} = 293 - \frac{(-700\ kJ)}{(4.76\ kg)(0.72\ \frac{kJ}{kg\ K})}$$

$$= 500\ K$$

Entropy balance

$$\Delta S = \int_1^2 \left(\cancel{\delta Q}^{0}/T\right) + \sigma$$

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$$\sigma = \Delta S = m(s_2 - s_1)$$

$$s_2 - s_1 = c_v \ln(T_2/T_1) + R\cancel{\ln(V_2/V_1)}_{0}$$

$$\Delta S = m(s_2 - s_1)$$

$$= m c_v \ln \frac{T_2}{T_1}$$

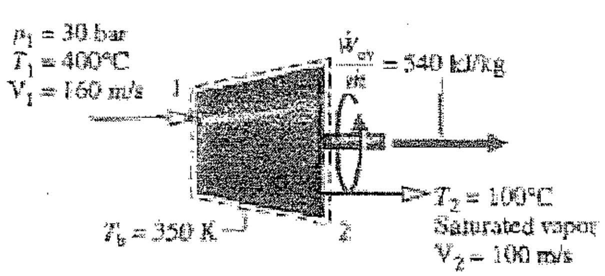
$$= (4.76)(0.72)\ln\left(\frac{500}{293}\right)$$

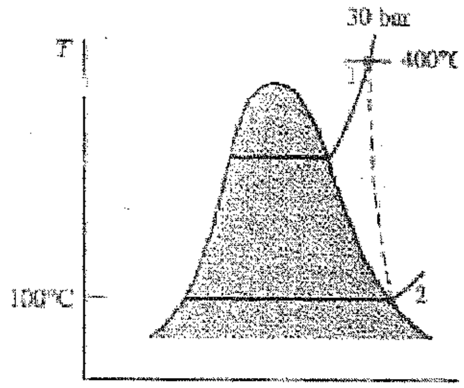
$$= 1.832 \text{ kJ/K}$$

Example: ||

Steam enters a turbine at 30 bar, 400 °C and at a velocity of 160 m/s. Saturated vapor leaves the turbine at 100 °C and at a velocity of 100 m/s. At steady state, $w_{turbine}$ = 540 kJ/kg of steam. Q from turbine to ambient occurs at 350 K. What is $\frac{\dot{\sigma}}{\dot{m}}$?

Answer:





Assumptions:

1. Steady state, 2. $\Delta Pe \approx 0$, 3. $\dot{m}_i = \dot{m}_e = \dot{m}$

Note: we don't take signs for $\dot{Q}$ and $\dot{W}_{cv}$ into consideration

The steady state equation is:

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

rearranging

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = \left(-\frac{\dot{Q}_{cv}/\dot{m}}{T_b}\right) + (s_2 - s_1)$$

To find $\dot{Q}_{cv}/\dot{m}$, apply an energy balance (1st Law):

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2}$$

<u>State 1:</u> From Table A-4

$$\left.\begin{aligned} P_1 &= 30\,\text{bar} \\ T_1 &= 400\,^\circ\text{C} \end{aligned}\right\} h_1 = 3230.9\,\text{kJ/kg}, \quad s_1 = 6.9212\,\text{kJ/kg}\cdot\text{K}$$

State 2: From Table A-2

$$\left.\begin{aligned} x_2 &= 1 \\ T_2 &= 100\ ^\circ\mathrm{C} \end{aligned}\right\} h_2 = 2676.1\,\mathrm{kJ/kg}, \quad s_2 = 7.3549\,\mathrm{kJ/kg\cdot K}$$

$$\frac{\dot{Q}_{cv}}{\dot{m}} = 540\frac{\mathrm{kJ}}{\mathrm{kg}} + (2676.1 - 3230.9)\frac{\mathrm{kJ}}{\mathrm{kg}} + \left(\frac{100^2 - 160^2}{2}\right)\frac{\mathrm{m}^2}{\mathrm{s}^2}\frac{\mathrm{N}}{\mathrm{kg\cdot m/s^2}}\frac{\mathrm{kJ}}{10^3\,\mathrm{N\cdot m}}$$

$$\frac{\dot{Q}_{cv}}{\dot{m}} = -22.6\frac{\mathrm{kJ}}{\mathrm{kg}}$$ [i.e heat is leaving CV]

Inserting values of $\frac{\dot{Q}_{cv}}{\dot{m}}$, s_2, s_1

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = -\frac{(-22.6\,\mathrm{kJ/kg})}{350\,\mathrm{K}} + (7.3549 - 6.9212)\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = 0.4983\,\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$$

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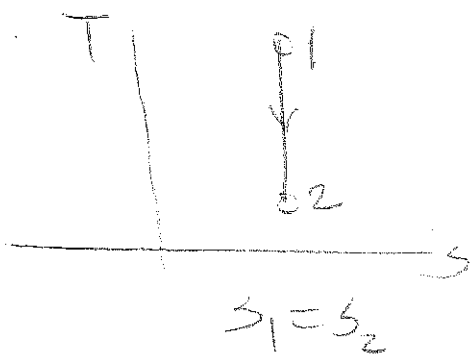
Example: 12

Air undergoes an isentropic process from $p_1 = 5$ bar at 500 K to $p_2 = 1$ bar. What is T_2?

Answer:



$$p_{r2} = \left(\frac{p_2}{p_1}\right)p_{r1}$$

From Table A-22 for 500 K, $p_{r1} = 8.411$

Hence,

$$p_{r2} = \left(\frac{1\,\text{bar}}{5\,\text{bar}}\right)(8.411)$$

$$p_{r2} = 1.6822$$

Interpolating using Table A-22 gives $T_2 = 317$ K

Method 2)

Assume c_p, c_v constant

$k = 1.4$ for air

$$\overset{0}{\cancel{s_2 - s_1}} = c_p \ln \frac{T_2}{T_1} - R\ln(P_2/P_1)$$

$$s_1 = s_2$$

$$T_2 = T_1 (P_2/P_1)^{\frac{k-1}{k}}$$

$$= 500\left(\frac{1}{5}\right)^{\frac{1.4-1}{1.4}} \cong 316\text{ K}$$

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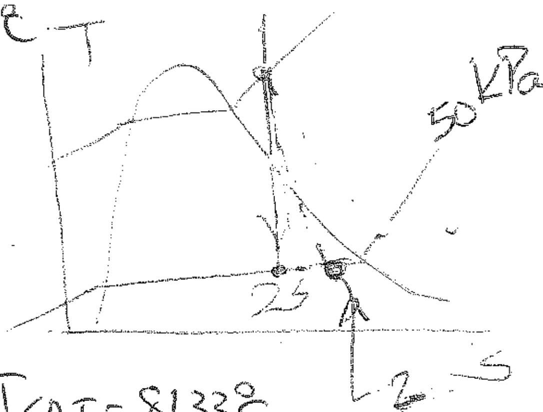
3) Steam enters an adiabatic turbine steadily at 3 MPa and 400°C and leaves at 50 kPa and 100°C. If the power output of the turbine is 2MW, determine:

a) the isentropic efficiency of the turbine,

b) the mass flow rate of the steam flowing through the turbine.

a)

State 1: $P_1 = 30$ bar $T_{SAT} = 233.9°C$



$$\left.\begin{array}{l} P_1 = 3\,\text{MPa} \\ T_1 = 400\,°\text{C} \end{array}\right\} h_1 = 3230.9\,\text{kJ/kg},\ s_1 = 6.9212\,\text{kJ/kg.K}$$

$T_1 > T_{SAT}$

State 2a:

$$\left.\begin{array}{l} P_{2a} = 50\,\text{kPa} \\ T_{2a} = 100\,°\text{C} \end{array}\right\} h_2 = 2682.5\,\text{kJ/kg}$$

$P_2 = 0.5$ bar $\rightarrow T_{SAT} = 81.33°C$

$T_2 > T_{SAT}$ superheat

The exit enthalpy of the steam for the isentropic process h_{2s} is determined from the requirement that the entropy of the steam remain constant ($s_{2s} = s_1$):

State 2S

$$\left.\begin{array}{l} P_{2s} = 50\,\text{kPa} \\ s_{2s} = s_1 \end{array}\right\} s_f = 1.0910\,\text{kJ/kg.K},\ s_g = 7.5939\,\text{kJ/kg.K}$$

Obviously, at the end of the isentropic process steam will exist as the saturated mixture since $s_f < s_{2s} < s_g$. Thus we need to find the quality at the state 2s first:

$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{6.9212 - 1.0910}{6.5029} = 0.897$$

and

$$h_{2s} = h_f + x_{2s} h_{fg} = 340.49 + 0.897(2305.4) = 2407.4\,\text{kJ/kg}$$

By substituting these enthalpy values into the isentropic efficiency equation:

$$\eta_t \cong \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{3230.9 - 2682.5}{3230.9 - 2407.4} = 0.666 \text{ or } 66.6\%$$

b) The mass flow rate of steam through this turbine is determined from the energy balance for steady-flow systems:

The power output of the turbine is determined from the rate form of the energy balance,

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i m_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e m_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

For steady state and one inlet one exit process, neglecting kinetic and potential energies:

$$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{m}(h_e - h_i)$$

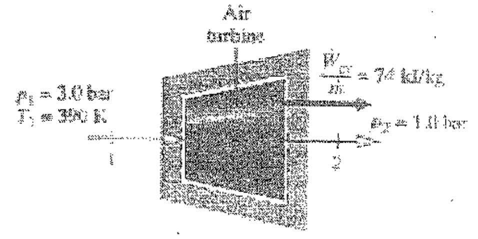
For adiabatic process :

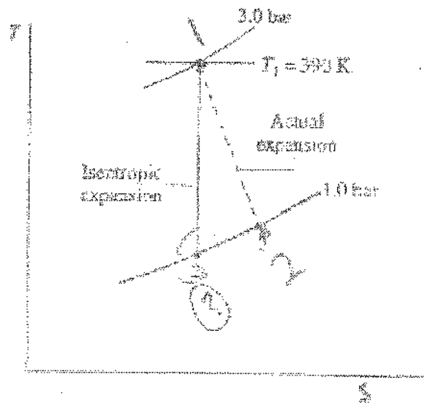
$$\Rightarrow \dot{W}_{cv} = \dot{m}(h_i - h_e) = \dot{m}(h_1 - h_2)$$

$$2\,\text{MW}\left(\frac{1000\,\text{kJ/s}}{1\,\text{MW}}\right) = \dot{m}(3230.9 - 2682.5)\,\text{kJ/kg}$$

$$\Rightarrow \dot{m} = 3.65\,\text{kg/s}$$

14) A turbine receives air at $p_1 = 3.0$ bar and $T_1 = 390$ K. Air leaves the turbine at $p_2 = 1.0$ bar. Work developed = 74 kJ/kg of air flow through turbine. The turbine operates adiabatically, $\Delta KE \cong 0$, $\Delta PE \cong 0$. What is the efficiency of the turbine, η_t ?





Assumption: The air is modeled as an ideal gas.

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_s = h_1 - h_{2s}$$

Isentropic expansion path

From table A-22 at 390 K, $h_1 = 390.88\ kJ/kg$

To find h_{2s}:

$$P_r(T_{2s}) = \left(\frac{P_2}{P_1}\right) P_r(T_1)$$

from Table A-22 at 390 K

$$P_r(T_{2s}) = \left(\frac{1.0\ bar}{3.0\ bar}\right)(3.481)$$

$$P_r(T_{2s}) = 1.1603$$

Interpolating in Table A-22 for this value of P_r gives

$$h_{2s} = 285.27\ kJ/kg$$

$$\therefore \left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_s = (390.88 - 285.27)\ kJ/kg = 105.6\ kJ/kg$$

$$\eta_t = \frac{\dot{W}/\dot{m}}{(\dot{W}/\dot{m})_s} = \frac{74\ kJ/kg}{105.6\ kJ/kg} = 0.70\ (or\ 70\%)$$

Method 2

$\left(\frac{\dot{W}t}{\dot{m}}\right)_s = (h_1 - h_2)$

Assume $c_p \Rightarrow$ const

$\left(\frac{\dot{W}t}{\dot{m}}\right)_s = c_p(T_1 - T_2)$

$T_1 = 390\,K$

$\frac{T_2}{T_1} = \left(P_2/P_1\right)^{\frac{k-1}{k}}$

$T_2 = 390\left(\frac{1}{3}\right)^{0.285} \cong 285\,K$

for air $c_p \approx 1.005$ kJ/kgK

$\left(\frac{\dot{W}t}{\dot{m}}\right)_s = (1.005)(390-285) \approx 105$ kJ/kg

$\eta_t = \frac{(\dot{W}/\dot{m})}{(\dot{W}/\dot{m})_s} = \frac{74}{105} = 0.704$ % = 70.4

(5) Air is compressed by an adiabatic compressor from 100 kPa and 12°C to a pressure of 800 kPa at a steady rate of 0.2 kg/s. If the isentropic efficiency of the compressor is 80%, determine:

c) the exit temperature of air,

d) the required power input to the compressor.

a) At the compressor inlet:

$$T_1 = 285\,\text{K} \Rightarrow h_1 = 285.14\,\text{kJ/kg}, Pr_1 = 1.1584$$

The enthalpy of the air at the end of the isentropic compression process is determined by using one of the isentropic relations of ideal gases,

$$P_{r2} = P_{r1}\left(\frac{P_2}{P_1}\right) = 1.1584\left(\frac{800\,\text{kPa}}{100\,\text{kPa}}\right) = 9.2672$$

and

$$Pr_2 = 9.2672 \Rightarrow h_{2s} = 517.05\,\text{kJ/kg}$$

Substituting the known quantities into the isentropic efficiency relation, we have

$$\eta_c \cong \frac{h_{2s} - h_1}{h_{2i} - h_1} \Rightarrow 0.80 = \frac{(517.05 - 285.14)\,\text{kJ/kg}}{(h_{2.} - 285.14)\,\text{kJ/kg}}$$

Thus,

$$h_{2i} = 575.03\,\text{kJ/kg} \Rightarrow T_{2i} = 569.5\,\text{K}$$

b) The required power input to the compressor is determined from the energy balance for steady-flow devices,

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i m_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum_e m_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

For steady state and one inlet one exit process, neglecting kinetic and potential energies:

$$\dot{Q}_{cv} - \dot{W}_{cv} = \dot{m}(h_e - h_i)$$

For adiabatic process:

?

$$\Rightarrow \dot{W}_{cv} = \dot{m}(h_i - h_e) = \dot{m}(h_1 - h_2)$$

$$\Rightarrow \dot{W}_{cv} = (0.2\,kg/s)(h_1 - h_{2a}) = (0.2\,kg/s)[(285.14 - 575.03)kJ/kg] = -58.0\,kW$$

Method 2

$P_1 = 100\,kPa$ $\quad$ $P_2 = 800\,kPa$

$T_1 = 12\,°C = 285\,K$

$$\text{so } \left(\frac{T_2}{T_1}\right)_s = (P_2/P_1)^{\frac{k-1}{k}} \Rightarrow T_{2s} = 285\left(\frac{800}{100}\right)^{0.285} = 515\,K$$

$$\eta_t = \frac{T_{2s} - T_1}{T_2 - T_1} \Rightarrow 0.8 = \frac{515 - 285}{T_2 - 285}$$

$$T_2 = 285 + \frac{515 - 285}{0.8} = 572\,K$$

b) $\cancel{\dot{Q}_{cv}}_0 - \dot{W}_{cv} = \dot{m}(h_2 - h_1)$

$$-\frac{\dot{W}_c}{\dot{m}} = (h_2 - h_1)$$

$\Rightarrow$ not choosing sign for compressor work

$\dot{W}_c = \dot{m}(h_1 - h_2)$

$\dot{W}_c = \dot{m}c_p(T_1 - T_2) \cong -\cancel{58}5\,kW$

$\dot{W}_c = (0.2)(1.005)(285 - 572)$

Work is done on compressor

16) . Steam at 7 MPa and 450°C is throttled in a valve to a pressure of 3 MPa during a steady flow process. Determine the entropy generated during this process and check if the increase of entropy principle is satisfied.

Noting that $h_2 = h_1$ for throttled process, the entropy of the steam at the inlet and the exit states is determined from the steam tables to be:

<u>State 1:</u>

$P_1 = 70$ bar $T_{sat} = 285°C$

$$\left.\begin{array}{l} P_1 = 7\,\text{MPa} \\ T_1 = 450\,°\text{C} \end{array}\right\} h_1 = 3287.1\,\text{kJ/kg},\ s_1 = 6.6327\,\text{kJ/kg.K}$$

<u>State 2:</u>

$$\left.\begin{array}{l} P_{2a} = 3\,\text{MPa} \\ h_2 = h_1 \end{array}\right\} s_2 = 7.0018\,\text{kJ/kg.K}$$

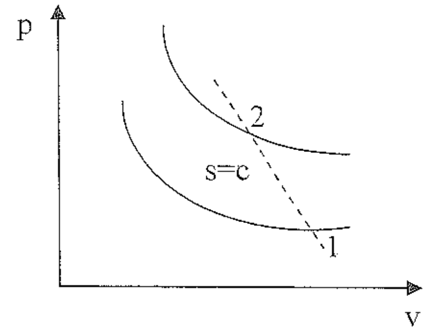
Then the entropy generation per unit mass of the steam is determined from the entropy balance applied to the throttling valve,

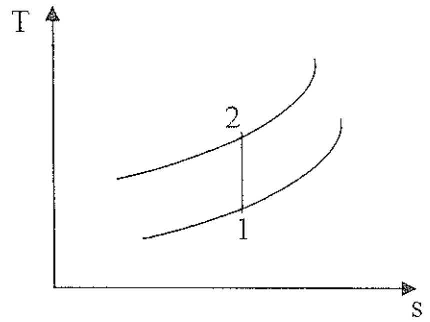
$$\frac{dS_{cv}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{\sigma}_{cv}$$

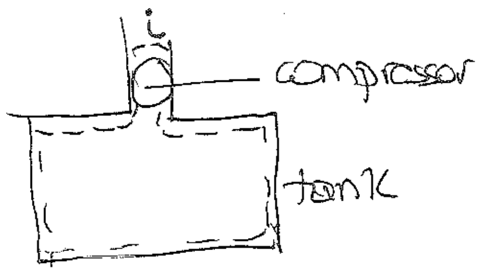
For steady flow and neglecting work and heat transfer with one inlet one outlet system:

$$\dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) \Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 = 7.0018 - 6.6327 = 0.3691\,\text{kJ/kg.K}$$

17) Assume an air tank has 40 L of 100 kPa air at ambient temperature 17°C. The adiabatic and reversible compressor is started so that it charges the tank up to a pressure of 1000 kPa and then it shuts off. We want to know how hot the air in the tank gets and the total amount of work required filling the tank.







$\Delta KE = 0$

$\Delta PE = 0$

CV: Compressor + tank

$$(m_2 - m_1)_{cv} + \sum \cancel{m_e}^{0} - \sum m_i = 0$$

$$(m_2 - m_1)_{cv} = m_i$$

$$\cancel{Q_{cv}}^{0} + \sum m_i h_i = \cancel{\sum m_e h_e}^{0} + [m_2 u_2 - m_1 u_1] + W_{cv}$$

$$m_i h_i = (m_2 u_2 - m_1 u_1) + W_{cv}$$

$$W_{cv} = m_1 u_1 - m_2 u_2 + (m_2 - m_1)_{cv} h_i$$

entropy balance

$$(m_2 s_2 - m_1 s_1) = \sum m_i s_i - \cancel{\sum m_e s_e}^{\,0} + \cancel{\int \frac{Q_{cv}}{T}}^{\,0} + \cancel{\sigma_{cv}}^{\,0}$$

$$(m_2 s_2 - m_1 s_1) = m_i s_i$$

$$m_2 s_2 = m_1 s_1 + m_i s_i$$

$$s_1 = s_i \Leftarrow \text{assume}$$

$$m_2 s_2 = (m_1 + m_i) s_1$$

$$= m_2 s_2$$

$$\therefore \quad s_2 = s_1$$

so $\cancel{s_2 - s_1} = s_2^o - s_1^o - R \ln (P_2/P_1)$

$$s_2^o = s_1^o + R \ln (P_2/P_1)$$

1) $T_1 = 17°C$, $P_1 = 100$ kPa $\Big\}$: $T_1 = 290K \rightarrow s_1^o = 1.66802 \ \frac{kJ}{kg\,K}$

$u_1 = \cancel{290.16}$ 206.91 $\frac{kJ}{kg}$

2) $P_2 = 1000$ kPa, $T_2 = ?$ | $s_2^o = s_1^o + R \ln (P_2/P_1)$

$$= 1.66802 + 0.287 \ln \left(\frac{1000}{100}\right)$$

$$= 2.3286 \ \text{kJ/kg K}$$

s_2^o	T (K)	u (kJ/kg)
2.31809	550	396.86
2.3286		
2.33685	560	404.42

$T_2 = 555.7 \; K$

$u_2 = 401.49 \; kJ/kg$

$$m_1 = \frac{P_1 V_1}{R T_1} = \frac{(100 \, kPa)(0.04 \, m^3)}{(0.287 \frac{kJ}{kg K})(290 K)} = 0.04806 \; kg$$

$$m_2 = \frac{P_2 V_2}{R T_2} = \frac{(1000 \, kPa)(0.04 \, m^3)}{(0.287 \frac{kJ}{kg K})(555.7 K)} = 0.2508 \; kg$$

Now At 290K $h_i = 290.16 \; kJ/kg$

so $W_{cv} = m_1 u_1 - m_2 u_2 + (m_2 - m_1) h_i$

$$W = (0.04806)(290.16) - (0.2508)(401.49)$$

$$+ (0.2508 - 0.04806)(290.16)$$

$$= -27.92 \; kJ$$