Çankaya University
Faculty of Engineering
Mechanical Engineering department
ME 211 Thermodynamics I
Midterm Exam 1
23.11.2017- SOLUTION

1) Nitrogen expands in a piston-cylinder system from 690 kPa and 260 ^{0}C to 210 kPa and 40 ^{0}C. Assuming ideal gas behavior, calculate the work done by the nitrogen. Assume pressure and volume are related by $p\forall^n = \text{const},$ where n is exponent.

$P_1 = 690 \text{ kPa}$
$T_1 = 260°C = 533 \text{ K}$
$P_2 = 210 \text{ kPa}$
$T_2 = 40°C = 313 \text{ K}$

$$P \cdot \forall^n = C \qquad P = \frac{C}{\forall^n}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$\Rightarrow \frac{313}{533} = \left(\frac{210}{690}\right)^{\frac{n-1}{n}}$$

$$\Rightarrow 0{,}5872 = 0{,}3043^{\frac{n-1}{n}} \quad \text{(take ln of both sides)}$$

$$\ln(0{,}5872) = \frac{n-1}{n} \cdot \ln(0{,}3043) \Rightarrow \frac{n-1}{n} = 0{,}447$$

$$\Rightarrow 0{,}447n = n - 1 \Rightarrow n = \frac{1}{0{,}5525} \Rightarrow n = 1.81$$

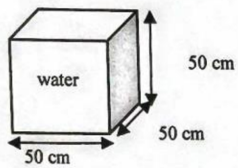
$$W = \int_{\forall_1}^{\forall_2} P d\forall = \int_{\forall_1}^{\forall_2} \frac{C}{\forall^n} d\forall = \frac{(P_2 \cdot \forall_2 - P_1 \cdot \forall_1)}{1-n}$$

$$R = \frac{\bar{R}}{M} = \frac{8{,}314}{28} = 0{,}2968 \frac{kJ}{kg}$$

From Ideal gas eqn : $P\forall = mRT$

$$w = \int_{T_1}^{T_2} \frac{R \cdot (T_2 - T_1)}{1-n} = \frac{0{,}2968 \cdot (313 - 533)}{1 - 1{,}81} = 80 \text{ kJ/kg}$$

2) A rigid cubic container, 50 cm on each side, contains a wet mixture of water vapor at $90\,^0C$ and 20 percent quality. Heat is added until the pressure is raised to 500 kPa. Determine the final temperature and the quantity of heat added.



$\forall = 0{,}125\,m^3$

$v = \frac{\forall}{m}$

State 1

$T_1 = 90°C\ ,\ x_1 = 0{,}2$ (Table A.2)

$v_f = 1{,}036 \cdot 10^{-3}\ m^3/kg$

$v_g = 2{,}361\ m^3/kg$

$v_1 = v_f + x(v_g - v_f)$

$\Rightarrow v_1 = 1{,}036 \cdot 10^{-3} + 0{,}2\,(2{,}361 - 1{,}036 \cdot 10^{-3})$

$\Rightarrow v_1 = 0{,}473\ m^3/kg$

$m = \frac{\forall}{v_1} = \frac{0{,}125}{0{,}473} = 0{,}2643\ kg$

$u_f = 376{,}85\ kJ/kg\ ,\ u_g = 2494{,}5\ kJ/kg$

$u_1 = u_f + x(u_g - u_f) = 376{,}85 + 0{,}2\,(2494{,}5 - 376{,}85)$

$u_1 = 800{,}38\ kJ/kg$

State 2 $P_2 = 500\ kPa,\ v_2 = v_1 = 0{,}473\ m^3/kg$

(Table A.3) $P = 5\ bar,\ v_f = 1{,}0926 \cdot 10^{-3}\ m^3/kg$

$v_g = 0{,}3749\ m^3/kg$

$v_2 > v_g \Rightarrow$ superheated!

(Table A-4) $P = 5\ bar$

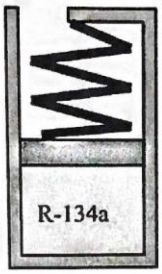
$T\ (°C)$	$v\ (m^3/kg)$	$u\ (kJ/kg)$
240	0,4646	2707,6
$T_2 = ?$	$v_2 = 0{,}473$	$u_2 = ?$
280	0,5034	2771,2

Interpolation: $T_2 \simeq 244°C$ and $u_2 = 2720.9\ kJ/kg$

1^{st} law: $\Delta U = {}_1Q_2 - {}_1W_2 \Rightarrow m(u_2 - u_1) = {}_1Q_2$ (with ${}_1W_2 = 0$)

${}_1Q_2 = 0{,}2643\,(2720{,}9 - 800{,}38) \rightarrow {}_1Q_2 = 507{,}6\ kJ$

3) A piston cylinder arrangement with a linear spring as shown in the figure contains R-134a at 12°C, x=0.4 and a volume of 0.02 m^3. It is then heated to 60 °C at which point specific volume is 0.04134 m^3/kg. Find the final pressure, the work and heat transfer in the process. Show the p-v diagram with respect to saturation lines.



State 1

$T_1 = 12°C$, $x = 0.4$ (sat. mix)

(Table A.10) $v_f = 0.7971 \cdot 10^{-3}$ m^3/kg

$v_g = 0.0460$ m^3/kg

$v_1 = v_f + x(v_g - v_f)$

$v_1 = 0.7971 \cdot 10^{-3} + 0.4(0.046 - 0.7971 \cdot 10^{-3})$

$\Rightarrow v_1 = 0.0192$ m^3/kg

$u_f = 65.83$ kJ/kg, $u_g = 233.63$ kJ/kg

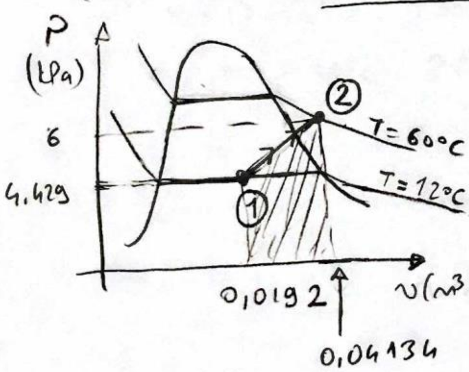
$u_1 = 65.83 + 0.4(233.63 - 65.83) = 132.95$ kJ/kg

$m = V_1 / v_1 = 0.02 / 0.0192 \Rightarrow m = 1.041$ kg

State 2

$T_2 = 60°C$ and $v_2 = 0.04134$ m^3/kg $\Rightarrow$ Superheated

(Table A.12) $P_2 = 6$ bar $= 600$ kPa, $u_2 = 273.54$ kJ/kg



$${}_1W_2 = \int_{V_1}^{V_2} P\,dV = m\int_{v_1}^{v_2} P\,dv$$

(Area of the trapezoid)

- $W = \frac{1.041}{2}\left[(600 + 442.9)(0.04134 - 0.0192)\right]$

$\Rightarrow {}_1W_2 = 12.02$ kJ

Energy balance:

$\Delta U = m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$ so $\Rightarrow {}_1Q_2 = m(u_2 - u_1) + {}_1W_2$

$\Rightarrow {}_1Q_2 = 1.041 \cdot (273.54 - 132.95) + 12.02$

$\Rightarrow {}_1Q_2 = 158.37$ kJ

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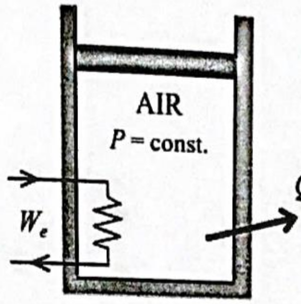
4.) A mass of 15 kg of air in a piston-cylinder device is heated from 25 ^{0}C to 77 ^{0}C by passing electric current through a resistance heater inside the cylinder. The pressure inside the cylinder is held constant at 300 kPa during the process , and a heat loss of 60 kJ occurs. Determine the electric energy supplied in kWh. (1 kWh=3600 kJ)



Energy balance:

$\Delta U = {}_1Q_2 - [W_e + W_b]$

${}_1Q_2 - W_e = m(u_2 - u_1) + W_b$.

where $W_b = \int_{V_1}^{V_2} P\,dV = P_2 V_2 - P_1 \cdot V_1$

$\Rightarrow {}_1Q_2 - W_e = \underbrace{m(u_2 - u_1) + m(P_2 v_2 - P_1 v_1)}_{m(h_2 - h_1)}$

$\Rightarrow {}_1Q_2 - W_e = m(h_2 - h_1)$

• 1st way: (variable cp)

(Table A.22) $T_1 = 25\,°C = 298\,K \Rightarrow h_1 = 298\,kJ/kg$.

$T_2 = 77\,°C = 350\,K \Rightarrow h_2 = 350{,}5\,kJ/kg$.

$\Rightarrow -60 - W_e = 15 \cdot (350{,}49 - 298)$

$\Rightarrow -W_e = 847{,}35 \Rightarrow W_e = -847{,}35\,kJ$

electrical energy $= -(-847{,}35) = 847{,}35\,kJ$.

$W_{elect} = 847{,}35\,kJ \left(\frac{kW\text{-}h}{3600\,kJ}\right) = 0{,}235\,kW\text{-}hr$.

• 2nd way: $T_{avg} = \frac{298 + 350}{2} \cong 325\,K \Rightarrow c_p = 1{,}0065\,kJ/kgK$

(const cp.) ${}_1Q_2 - W_e = m \cdot c_p \cdot (T_2 - T_1)$

$-60 - W_e = 15 \cdot 1{,}0065\,(350 - 298)$

$W_e = -845.1\,kJ$. , $W_{elect} = 0{,}235\,kW\text{-}hr$