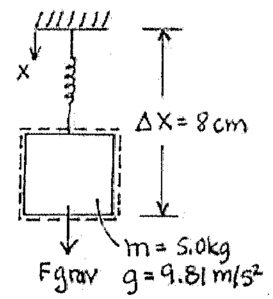
CHAPTER 1

EXAMPLES

1. When an object of mass 5 kg is suspended from a spring, the spring is observed to stretch by 8 cm. The deflection of the spring is related linearly to the weight of the suspended mass. What is the proportionality constant, in newton per cm, if g = 9.81 m/s^2?

PROBLEM 1.11



$F_{spring} = K(\Delta X)$ and $F_{spring} = F_{grav} = mg$

Thus

$K(\Delta x) = mg$

and

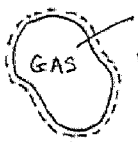
$$K = \frac{mg}{\Delta X}$$

$$= \frac{(5.0\ kg)(9.81\ m/s^2)}{(8\ cm)} \left| \frac{1\ N}{kg \cdot m/s^2} \right|$$

$$= 6.131\ N/cm \longleftarrow \text{proportionality constant}$$

2. A closed system consisting of 5 kg of a gas undergoes a process during which the relationship between pressure and specific volume is $pv^{1.3}$ = constant. The process begins with $p_1 = 1$ bar, $v_1 = 0.2\ m^3/kg$ and ends with $p_2 = 0.25$ bar. Determine the final volume, in m^3, and plot the process on a graph of pressure versus specific volume.

PROBLEM 1.27



$m = 5\,kg$

$pv^{1.3} = \text{const.}$

$p_1 = 1$ bar, $v_1 = 0.2\ m^3/kg$

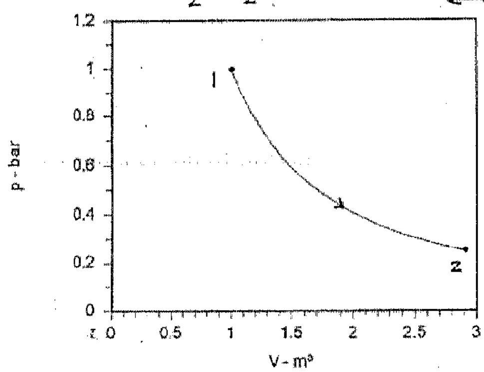
$p_2 = 0.25$ bar

From the pressure-specific volume relation

$$v_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.3}} v_1 = \left(\frac{1}{0.25}\right)^{\frac{1}{1.3}} (0.2\ m^3/kg)$$

$$= 0.5810\ m^3/kg$$

$$V_2 = v_2 m = 2.905\ m^3 \longleftarrow V_2$$



3. A gas contained within a piston-cylinder assembly undergoes a thermodynamic cycle consisting of three processes:

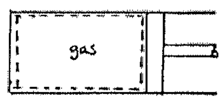
<u>Process 1-2:</u> Compression with pV = constant from $p_1 = 1$ bar, $V_1 = 1.0\ m^3$ to $V_2 = 0.2\ m^3$

<u>Process 2-3:</u> Constant-pressure expansion to $V_3 = 1.0\ m^3$

<u>Process 3-1:</u> Constant volume

Sketch the cycle on a p-V diagram labeled with pressure and volume values at each numbered state.

PROBLEM 1.31



Three processes:

1-2: pV = constant
$p_1 = 1$ bar, $V_1 = 1\ m^3$, $V_2 = 0.2\ m^3$

2-3 p = constant, $V > V_2$ (expansion), $V_3 = 1.0\ m^3$

3-1 V = constant

For process 1-2, pV = constant. The constant can be evaluated using data at state 1:

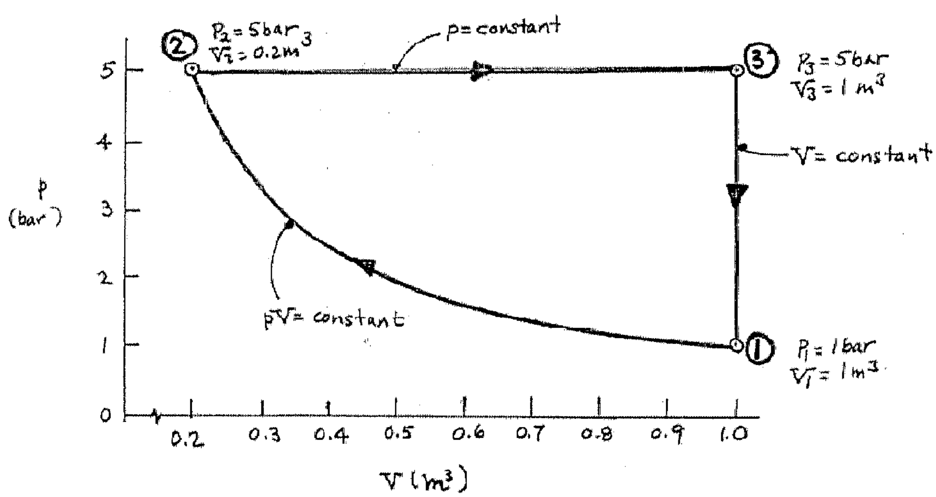
$$pV = \text{constant} = p_1 V_1 = (1\ \text{bar})(1\ m^3) = 1\ \text{bar}\cdot m^3$$

Accordingly, on a pressure-volume plot process 1-2 is described by

$$p = \frac{1\ \text{bar}\cdot m^3}{V}$$

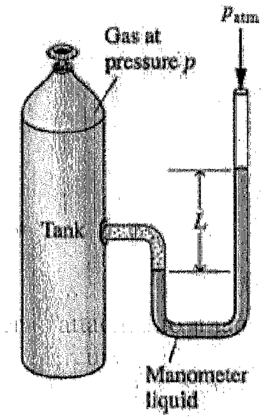
In particular, when $V_2 = 0.2\ m^3$, $p = 5$ bar.

Sketching the processes in series,



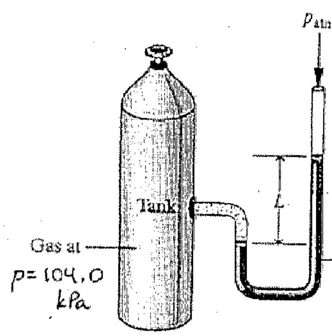
4. As shown in figure, a manometer is attached to a tank of gas in which the pressure is 104.0 kPa. The manometer liquid is mercury, with a density of 13.59 g/cm^3. If g = 9.81 m/s^2 and the atmospheric pressure is 101.33 kPa, calculate

a) the difference in mercury levels in the manometer, in cm.
b) the gage pressure of the gas, in kPa.



Solution

PROBLEM 1.32



$$P - P_{atm} = \rho g L$$

$\rho = 13.59\ g/cm^3$
$g = 9.81\ m/s^2$

(a)

$$L = \frac{P - P_{atm}}{\rho g}$$

$$= \frac{(104.0 - 101.33)\ kPa}{(13.59\ g/cm^3)(9.81\ m/s^2)} \left|\frac{10^3 N/m^2}{1\ kPa}\right| \left|\frac{10^3 g}{1\ kg}\right| \left|\frac{1\ m^3}{10^6\ cm^3}\right| \left|\frac{1\ kg \cdot m/s^2}{1 N}\right|$$

$= 0.02\ m = 2\ cm$ ← L

(b) $P_{gage} = P - P_{atm} = 104.0 - 101.33 = 2.67\ kPa$ ← P_{gage}

5. A closed system consists of 0.5 kmol of ammonia occupying a volume of 6 m^3. Determine (a) the weight of the system in N, (b) the specific volume in m^3/kmol and m^3/kg. Let g=9.81 m/s^2.

PROBLEM 1.21

0.5 Kmol NH_3, $V = 6m^3$

$g = 9.81 m/s^2$

(a) $F_{grav} = m\,g$

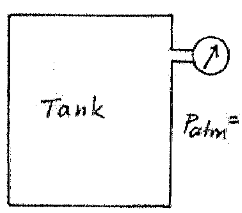
Using Eq. 1.8, $m = n\,M = 0.5\,\text{kmol}\left(17.03\,\frac{\text{kg}}{\text{kmol}}\right) = 8.52\,\text{kg}$ ← Table A-1

$\therefore\; F_{grav} = (8.52\,\text{kg})(9.81\,\frac{m}{s^2})\left|\frac{1N}{1\,\text{kg}\cdot m/s^2}\right| = 83.58N$ ← F_{grav}

(b) $\bar{v} = \frac{V}{n} = \frac{6m^3}{0.5\text{kmol}} = 12\,\frac{m^3}{\text{kmol}}$, $v = \frac{V}{m} = \frac{6m^3}{8.52\text{kg}} = 0.704\frac{m^3}{kg}$ ← $\bar{v}, v$

6. The absolute pressure inside a tank is 0.4 bars, and the surrounding atmospheric pressure is 98 kPa. What reading would a Bourdon gage mounted in the tank wall give, in kPa? Is this a gage or vacuum reading?

PROBLEM 1.34



$P_{atm} = 98$ kPa

$P_{abs} = 0.4$ bar $= 40$ kPa

$P_{abs} < P_{atm} \Rightarrow$ vacuum

$P_{vac} = P_{atm} - P_{abs}$

$= 98 - 40 = 58$ kPa ← P_{vac}

7. Figure below shows a tank within a tank, each containing air. The absolute pressure in tank A is 267.7 kPa. Pressure gage A is located inside tank B and reads 1.4 bars. The U-tube manometer connected to tank B contains mercury. Using data on the diagram determine the absolute pressures inside tank B in kPa and the column length L in cm. The atmospheric pressure surrounding tank B is 101 kPa. The acceleration of gravity is $g = 9.81\ m/s^2$.

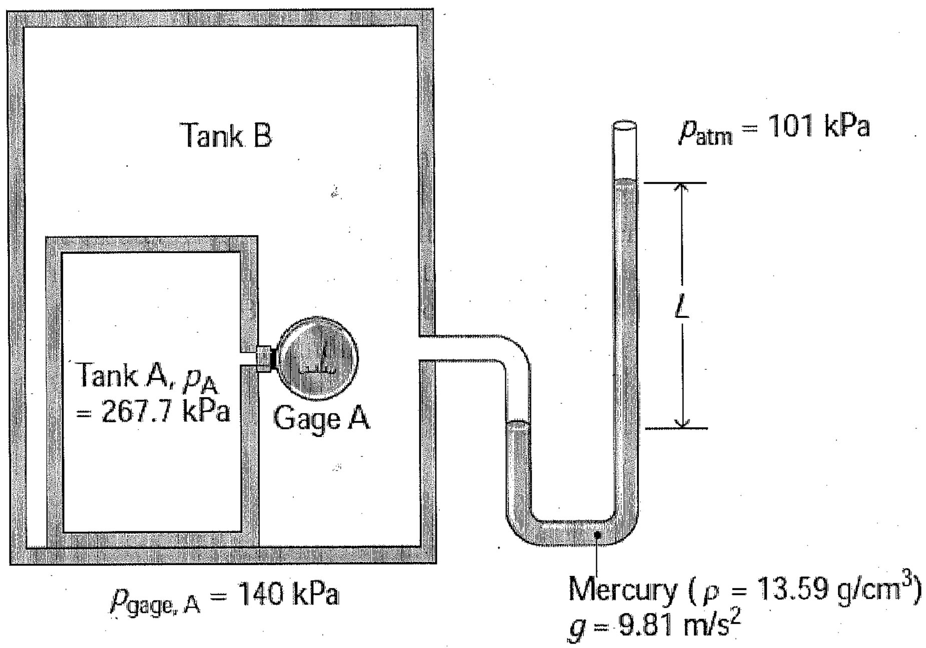


Fig. P1.37

Solution

continued (7)

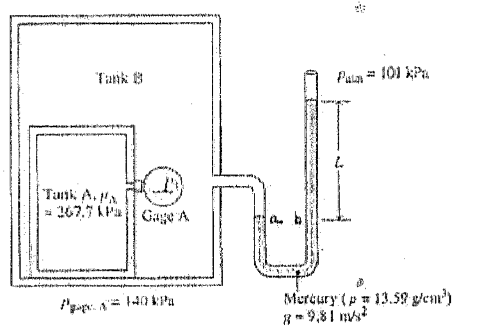


Fig. P1.37

Looking at gage A,

$$P_{gage,A} = P_A - P_B$$

$$\Rightarrow \quad P_B = P_A - P_{gage,A}$$
$$= 267.7\,kPa - 140\,kPa$$
$$= 127.7\,kPa$$

The pressures at a and b are equal. At a, the pressure is P_B. At b, the pressure is

$$P_b = P_{atm} + \rho g L$$

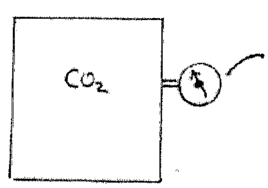
So, $\quad P_B = P_{atm} + \rho g L$

Solving,

$$L = \frac{P_B - P_{atm}}{\rho g} = \frac{(127.7\,kPa - 101\,kPa)\left|\frac{10^3\,N/m^2}{1\,kPa}\right|\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right|}{\left(13.59\,\frac{g}{cm^3}\right)\left|\frac{1\,kg}{10^3\,g}\right|\left|\frac{10^2\,cm}{1\,m}\right|^3\left(9.81\,\frac{m}{s^2}\right)} = 0.2\,m$$

$$= (0.2\,m)\left|\frac{10^2\,cm}{1\,m}\right| = 20\,cm$$

8. A vacuum gage indicates that the pressure of carbondioxide in a closed chamber is -10 kPa. The pressure of the surrounding atmosphere is equivalent to a 750-mm column of mercury. The density of mercury is 13.59 g/cm^3, and the acceleration of gravity is 9.81 m/s^2. Determine the absolute pressure of CO_2 within the chamber, in kPa.



P_{atm} = 750 mmHg, ρ_m = 13.59 g/cm³, g = 9.81 m/s²

P_{gage} = 10 kPa (vacuum)

The atmospheric pressure, in kPa, can be found using the "for example" of Sec. 1.6.1. The calculation is the same as used there, giving $P_{atm} = 10^5\,N/m^2$

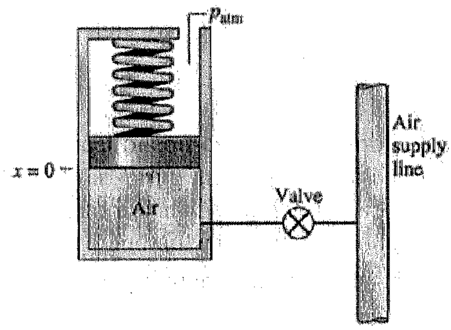
Thus, $P_{atm} = 10^5\,\frac{N}{m^2}\left|\frac{1\,kPa}{10^3\,N/m^2}\right| = 100\,kPa$. Then, with Eq. 1.15

$$P_{CO_2} = 100\,kPa - 10\,kPa = 90\,kPa$$

9. Air contained within a vertical piston-cylinder assembly is shown in figure. On its top, the 10-kg piston is attached to a spring and exposed to an atmospheric pressure of 1 bar. Initially, the bottom of the piston is at x = 0, and the spring exerts a negligible force on the piston. The valve is opened and air enters the cylinder from the supply line, causing the volume of the air within the cylinder to increase by 3.9×10^{-4} m^3. The force exerted by the spring as the air expands within the cylinder varies linearly with x according to

$F_{spring} = kx$

where k = 10,000 N/m. The piston face area is 7.8×10^{-3} m^2. Ignoring friction between the piston and the cylinder wall, determine the pressure of the air within the cylinder, in bar, when the piston is in its initial position. Repeat when the piston is in its final position. The local acceleration of gravity is 9.81 m/s^2.



Solution

see next page

continued

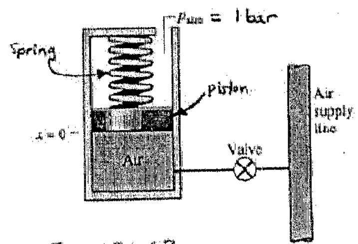


Figure P1.43

$g = 9.81\ m/s^2$

$F_{spring} = kx$, where $k = 10{,}000\ N/m$

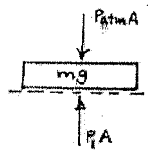
For the piston:

$m = 10\ kg$

$A = 7.8 \times 10^{-3}\ m^2$

For the air: $\Delta V = 3.9 \times 10^{-4}\ m^3$

Initially, x=0 and there is no spring force acting on the piston. Also, friction between the piston and the cylinder wall can be ignored. Accordingly, the force exerted by the air within the cylinder on the bottom of piston is equal to the weight of the piston plus the force exerted by the atmosphere on the top of the piston:



$\Sigma F_x = 0$:

$$P_1 A = P_{atm} A + mg$$

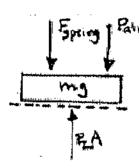
$$P_1 = P_{atm} + \frac{mg}{A}$$

$$P_1 = 1\ bar + \left[\frac{(10\ kg)(9.81\ m/s^2)}{7.8 \times 10^{-3}\ m^2}\right]\left|\frac{1\ N}{1\ kg \cdot m/s^2}\right|\left|\frac{1\ bar}{10^5\ N/m^2}\right|$$

$$P_1 = 1.126\ bar \longleftarrow P_1$$

Finally, the force exerted by the air within the cylinder on the bottom of the piston is equal to the weight of the piston plus the force exerted by the atmosphere on the top of the piston plus the force exerted by the spring on the top of the piston:

Problem 1-43 continued



Ignoring friction,

$\Sigma F_x = 0$:

$$P_2 A = P_{atm} A + mg + F_{spring}$$

$$P_2 = P_{atm} + \frac{mg}{A} + \frac{F_{spring}}{A}$$

For the spring, $F_{spring} = kx$, where x is found using the increase in volume of the air: $x = \frac{\Delta V}{A} = \frac{3.9 \times 10^{-4}\ m^3}{7.8 \times 10^{-3}\ m^2} = 0.05\ m$.

Collecting results

$$P_2 = P_{atm} + \frac{mg}{A} + \left[\frac{(10{,}000\ N/m)(0.05\ m)}{(7.8 \times 10^{-3}\ m^2)}\left|\frac{1\ bar}{10^5\ N/m^2}\right|\right]$$

$$= 1.126\ bar + 0.641\ bar$$

$$= 1.767\ bar \longleftarrow P_2$$

10) solution

$$T = a \ln(P) + b$$

ice point $T = 0°C$ $P = 1.86$

steam point $T = 100°C$ $P = 6.81$

so substitute in equation

$$\left.\begin{array}{l} 0 = a \ln(1.86) + b \\ 100 = a \ln(6.81) + b \end{array}\right]$$

so

$a = 77.0525$ $b = -47.8169$

now substitute a, and b into equation

$$T = 77.0525 \ln(P) - 47.8169$$

Now when $P = 2.5$ $T = 22.78°C$

see next page for calculation using Maple.

```
> restart :
> with(linalg) :
> eq1 := a·ln(1.86) + b = 0;
                    eq1 := 0.6205764877 a + b = 0                    (1)
>
> eq2 := a·ln(6.81) + b = 100;
                    eq2 := 1.918392120 a + b = 100                   (2)
>
> sol := solve({eq1, eq2}, {a, b});
            sol := {a = 77.05254700, b = -47.81699898}               (3)
> a := op(2, sol[1]);
                    a := 77.05254700                                 (4)
> b := op(2, sol[2]);
                    b := -47.81699898                                (5)
> T := a·ln(P) + b;
            T := 77.05254700 ln(P) - 47.81699898                     (6)
> evalf(subs(P = 2.5, T));
                    22.78553571                                      (7)
>
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