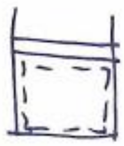
Çankaya University
Faculty of Engineering
Mechanical Engineering Department
ME 211 Thermodynamics I
Midterm Exam 2 Solution
22.12.2017

Q-1 Air at 1 atm and 20 ^{0}C occupies an initial volume of 1000 cm^3 in a cylinder. The air is confined by a piston which has a constant restraining force so that the gas pressure always remains constanyt. Heat is added to the air until its temperature reaches 260 ^{0}C. Calculate (a) the heat added (b) the work done by the gas, (c) the change in internal energy of the gas.



$P_1 = 1\,atm$
$T_1 = 20°C$
$V_1 = 1000\,cm^3$

$T_2 = 260°C$
$P_2 = P_1 = 1\,atm$

$Q - W = U_2 - U_1 = m(u_2 - u_1)$
$W_b = \int p\,dV = pm(v_2 - v_1)$

$Q = m(u_2 - u_1) + p(v_2 - v_1)m = m(h_2 - h_1)$

$P_1 V_1 = mRT_1 \Rightarrow m = \frac{P_1 V_1}{RT_1}$

$R = \frac{\bar{R}}{M} = \frac{8314\ kJ/kmol.K}{28.97\ kg/kmol} = 287\ \frac{J}{kg \cdot K}$

$m = \frac{P_1 V_1}{RT_1} = \frac{(1.0325 \times 10^5\ N/m^2)(1000\,cm^3 \times 10^{-6}\,m^3/cm^3)}{(287\ \frac{J}{kmol\,K})(293\,K)}$

$= 1.205 \times 10^{-3}\ kg$

Assume cp constant

$Q = mc_p(T_2 - T_1)$
$= (1.205 \times 10^{-3}\,kg)(1.005\ \frac{kJ}{kg\,K})(260 - 20)$ (533 K, 293 K)
$= 290.6\ J$

b) $W = pm(v_2 - v_1)$

$P_1 V_1 = mRT_1$
$P_2 V_2 = mRT_2$ $\Rightarrow \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$
$P_1 = P_2$

$$\therefore V_2 = T_2\left(\frac{V_1}{T_1}\right) = V_1\left(\frac{T_2}{T_1}\right)$$

$$= (1000\ cm^3)\left(\frac{533}{293}\right) = 1819\ cm^3$$

$$W_b = \left(1.0325\times10^5 \frac{N}{m^2}\right)(1819 - 1000)cm^3 \cdot \left(\frac{1m}{100cm}\right)^3$$

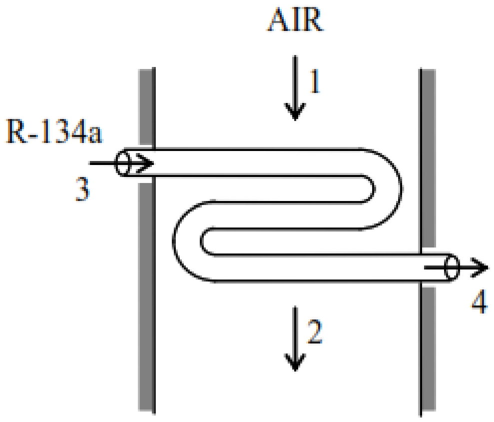
$$= 82{,}98\ J$$

$$Q - W = \Delta U$$

$$\Delta U = Q - W = 290.6 - 82{,}98$$

$$= 207{,}6\ J$$

Q-2 Refrigerant 134-a at 1 MPa and 90 ^{0}C is to be cooled to 1 MPa and 30^{0}C in a condenser by air. The air enters at 100 kPa and 27 ^{0}C with a volume flow rate of 600 m^3/min and leaves at 95 kPa and 60 ^{0}C. Determine the mass flow rate of the refrigerant.



air → $R = 0.287$ kJ/kg K

• Mass balance:

air: $\dot{m}_1 = \dot{m}_2 = \dot{m}_a$ R-134a: $\dot{m}_3 = \dot{m}_4 = \dot{m}_R$

• energy balance:

$$\dot{Q}_{cv} - \dot{W}_{cv} = \sum \dot{m}_e \cdot h_e - \sum \dot{m}_i \cdot h_i \quad (\Delta Ke = \Delta Pe = 0)$$

($\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$)

$$\Rightarrow \dot{m}_2 \cdot h_2 + \dot{m}_4 \cdot h_4 = \dot{m}_1 \cdot h_1 + \dot{m}_3 \cdot h_3$$

$$\Rightarrow \dot{m}_a h_2 + \dot{m}_R \cdot h_4 = \dot{m}_a \cdot h_1 + \dot{m}_R h_3$$

$$\Rightarrow \dot{m}_a (h_2 - h_1) = \dot{m}_R (h_3 - h_4) \Rightarrow \dot{m}_R = \frac{\dot{m}_a (h_2 - h_1)}{(h_3 - h_4)}$$

State 1: air

$P_1 = 100\,kPa$

$T_1 = 27°C = 300\,K$

Table A.22 $h_1 = 300{,}19\ kJ/kg$

State 2: air

$P_2 = 95\,kPa$

$T_2 = 60°C$

Table A.22 $h_2 = 333{,}4\ kJ/kg$

State 3: R-134a

$P_3 = 10\,bar$

$T_3 = 90°C$

Superheated
From table A.12
$h_3 = 324{,}01\ kJ/kg$

State 4: R-134a

$P_4 = 10\,bar$

$T_4 = 30°C$

10 bar → $T_{sat} = 39{,}39°C$
→ compressed liquid
Table A.10 → $h_4 = h_f = 91{,}49\ kJ/kg$

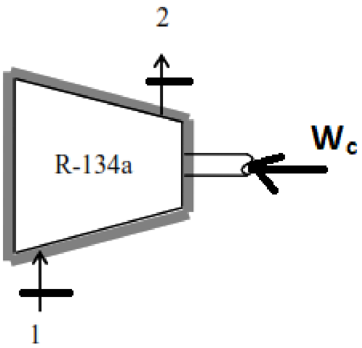
$\dot{V}_1 = 600\,m^3/min = \frac{600}{60} = 10\,m^3/s$

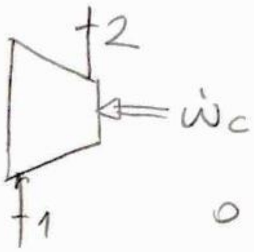
$\dot{m}_a = \frac{\dot{V}_1}{v_1}$ where $v_1 = \frac{R \cdot T_1}{P_1} = \frac{0{,}287 \cdot 300}{100} = 0{,}861\,m^3/kg$

$\Rightarrow \dot{m}_a = \frac{10}{0{,}861} = 11.61\,kg/s$

$\dot{m}_R = \frac{11{,}61 \cdot (333{,}4 - 300{,}19)}{(324{,}01 - 91{,}49)} = 1.6588\,kg/s = 99.5\,kg/min$

Q-3 Refrigerant 134-a enters an adiabatic compressor as saturated vapor at -24 ^{0}C and leaves at 0.8 MPa and 60 ^{0}C. The mass flow rate of the refrigerant is 1.2 kg/s. Determine a)the power input of the compressor b) the volume flow rate of the refrigerant at the compressor inlet.





$\Delta ke = \Delta Pe = 0$

$Q = 0$ (adiabatic)

$\dot{Q}_{cv} - \dot{W}_{cv} = \sum \dot{m}_e h_e - \sum \dot{m}_i \cdot h_i \quad (\dot{m}_i = \dot{m}_e)$ ($\dot{Q}_{cv} = 0$)

$\Rightarrow -\dot{W}_c = \dot{m}(h_2 - h_1)$

State 1 : Sat vapor, $T_1 = -24°C$, From table A.10 → $h_1 = 19.29$ kJ/kg, $v_1 = 0.7296 \cdot 10^{-3}$ m³/kg

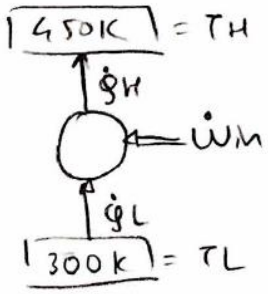
State 2 : $P_2 = 0.8$ MPa, $T_2 = 60°C$ → superheated, $h_2 = 294.98$ kJ/kg

$-\dot{W}_{cv} = 1.2 \cdot (294.98 - 19.29) = -330.8$ KW

$\dot{m} = \frac{\dot{V}}{v} \Rightarrow \dot{V} = 1.2 \cdot 0.7296 \cdot 10^{-3} = 0.000875$ m³/s

Q-4 At steady state, the power input of a refrigeration cycle is 500 kW. The cycle operates between hot and cold reservoirs which are at 550 K and 300 K, respectively.

a) If cycle's coefficient of performance is 1.6, determine the rate of energy removed from cold reservoir, in kW.
b) Determine the minimum theoretical power required, in kW, for any such cycle operating between 550 K and 300 K.



$$C.O.P = \beta = 1.6 = \frac{\dot{Q}_L}{\dot{W}_M} = \frac{\dot{Q}_L}{500} \Rightarrow \dot{Q}_L = 800 \text{ kW}$$

Max amount of rate of energy ($\dot{Q}_{Lmax}$) → reversible cycle (CARNOT CYCLE)

$$\beta_{max} = COP_{max} = \frac{T_L}{T_H - T_L} = \frac{300}{450-300} = 2$$

then $$COP = \beta_{max} = \frac{\dot{Q}_{Lmax}}{\dot{W}_M} = \frac{\dot{Q}_{Lmax}}{500}$$

$$\Rightarrow \dot{Q}_{Lmax} = 1000 \text{ kW}$$