ÇANKAYA UNIVERSITY
DEPARTMENT OF MECHANICAL ENGINEERING
ME211 THERMODYNAMICS I
Homework #1 Solutions

1) A closed system consists of **0.2 kmol** of ammonia occupying a volume of 0.3 m^3. Determine:
   a) The weight of the system in N
   b) The specific volume in m^3/kmol and m^3/kg. Note: $g=9.81\ m/s^2$.

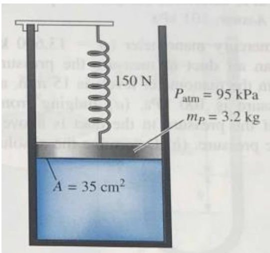
Solution:

For Amonia,From Table A.1, M= 17.03 kg/kmol

a) m= 0.2*17.03= 3.406 kg.v
   Weight→ F=m*g=3.406*9.81=33.41 N

b) υ=V/m=0.3/3.406=0.088 m^3/kg
   $\bar{\upsilon} = \frac{V}{n} = \frac{0.3}{0.2} = 1.5\mathrm{m}^3/\mathrm{kmol}$

2) A gas is contained in a vertical, frictionless piston-cylinder device. The piston has a mass of **3.2 kg** and a cross-sectional area of **35 cm^2**. A compressed spring above the piston exerts a force of **150 N** on the piston. If atmospheric pressure is **95 kPa**, determine the pressure inside the cylinder.



Solution:

Analysis Drawing the free body diagram of the piston and balancing the vertical forces yield

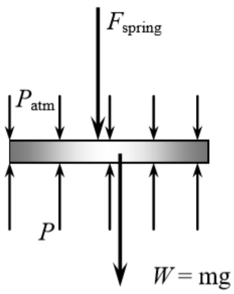
$$PA = P_{atm}A + W + F_{spring}$$

Thus,

$$P = P_{atm} + \frac{mg + F_{spring}}{A}$$

$$= (95\ \mathrm{kPa}) + \frac{(3.2\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) + 150\ \mathrm{N}}{35 \times 10^{-4}\ \mathrm{m}^2}\left(\frac{1\ \mathrm{kPa}}{1000\ \mathrm{N/m^2}}\right)$$

$$= \mathbf{147\ kPa}$$



3) A gas is compressed from an initial volume 0f **0.42 m³** to a final volume of **0.12 m³**. During the quasi-equilibrium process, the pressure changes with volume according to the relation **P=aV+b**, where **a=-1200 kPa/m³** and **b= 600 kPa**. Calculate the work done during this process;
   a) By plotting the process on P-V diagram and finding the area under the process curve.
   b) By performing the necessary integration.

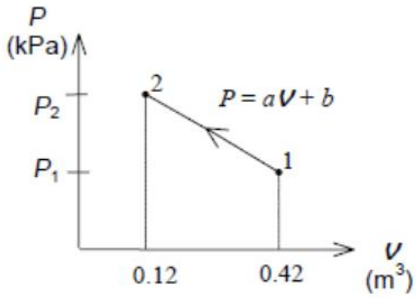
Solution:

Assumptions The process is quasi-equilibrium.

Analysis (*a*) The pressure of the gas changes linearly with volume, and thus the process curve on a *P-V* diagram will be a straight line. The boundary work during this process is simply the area under the process curve, which is a trapezoidal. Thus,

$$P_1 = aV_1 + b = (-1200 \text{ kPa/m}^3)(0.42 \text{ m}^3) + (600 \text{ kPa}) = 96 \text{ kPa}$$

$$P_2 = aV_2 + b = (-1200 \text{ kPa/m}^3)(0.12 \text{ m}^3) + (600 \text{ kPa}) = 456 \text{ kPa}$$



and

$$W_{b,\text{out}} = \text{Area} = \frac{P_1 + P_2}{2}(V_2 - V_1)$$

$$= \frac{(96 + 456)\text{kPa}}{2}(0.12 - 0.42)\text{m}^3 \left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3}\right)$$

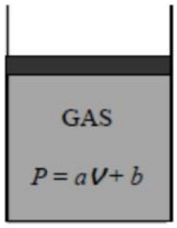
$$= \mathbf{-82.8 \text{ kJ}}$$

(*b*) The boundary work can also be determined by integration to be

$$W_{b,\text{out}} = \int_1^2 P\,dV = \int_1^2 (aV + b)\,dV = a\frac{V_2^2 - V_1^2}{2} + b(V_2 - V_1)$$

$$= (-1200 \text{ kPa/m}^3)\frac{(0.12^2 - 0.42^2)\text{m}^6}{2} + (600 \text{ kPa})(0.12 - 0.42)\text{m}^3$$

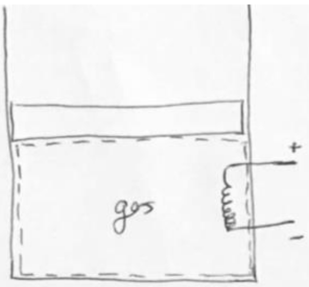
$$= \mathbf{-82.8 \text{ kJ}}$$



Discussion The negative sign indicates that work is done on the system (work input).

4) A vertical piston–cylinder assembly by a piston with a face area of **80 cm²** and weight of **4450 N** contains a gas. The atmosphere exerts a pressure of **101.1 kPa** on top of the piston. An electrical resistor transfers energy to the gas in the amount of **6 kJ** as the elevation of the piston increases by **0.7 m**. The piston and cylinder are poor thermal conductors, and friction between them can be neglected. Determine the change in internal energy of the gas, in **KJ**.

Solution:



$A_P = 80\ cm^2$

$W_p = 4450\ N$

$P_{atm} = 101.1\ kPa$

$W_e = 6\ kJ$

$\Delta z)_p = 0.7\ m$

- piston and cylinder are poor conductors.
- Friction between piston and cylinder.

FIND:

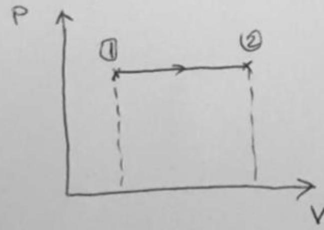
$\Delta U)_{gas} = ?$

Assumptions:

1) Closed system consists of gas and resistor
2) Kinetic energy change of gas is zero
3) Potential energy change of piston is negligible
4) Since cylinder and piston are poor conductor, heat transfer to ~~the~~ may be ignored.
5) An electric work is done by the resistor.

Analysis:

Process is a constant pressure expansion process.



Energy balance equation for the system

$\Delta E = Q - W$

$(\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U)_{gas} = \cancel{Q} - W$
0 0 0

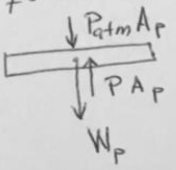
$\therefore \Delta U)_{gas} = -W$

Work consist of moving boundary work and electric work, i.e.

$W = W_b - W_e$

$W_b = \int_1^2 P dV = P \Delta V$

P can be determined from the force balance of piston:



$\Rightarrow P A_p = P_{atm} A_p + W_p$

$P = P_{atm} + \frac{W_P}{A_P}$

$P = 101100 \left[\frac{N}{m^2}\right] + \frac{4450 [N]}{80 \times 10^{-4} [m^2]} = 657350 \left(\frac{N}{m^2}\right)$

Volume change ΔV can be obtained from the displacement of piston, i.e.

$\Delta V = A_P \Delta z = 80 \times 10^{-4} [m^2] \times 0.7 [m]$

$\boxed{\Delta V = 56 \times 10^{-4}\ m^3}$

$W_b = P \Delta V = 657350 \left[\frac{N}{m^2}\right] \times 56 \times 10^{-4}\ m^3$

$W_b = 3681.16\ J = 3.68116\ kJ.$

$W_e = 6\ kJ$ given.

$\therefore \Delta U)_{gas} = -(W_b - W_e) = -(3.68116 - 6)$

$= 2.32\ kJ$

5) A gas within a piston–cylinder assembly undergoes a thermodynamic cycle consisting of three processes (There are no significant changes in kinetic or potential energy):
Process 1–2: Constant volume, $V = 0.028\ m^3$, $U_2 - U_1 = 26.4$ kJ.
Process 2–3: Expansion with pV= constant, $U_3 = U_2$.
Process 3–1: Constant pressure, $p = 1.4$ bar, $W_{31} = -10.5$ kJ.
(a) Sketch the cycle on a p–V diagram.

(b) Calculate the net work for the cycle, in kJ.
(c) Calculate the heat transfer for process 2–3, in kJ.
(d) Calculate the heat transfer for process 3–1, in kJ.
(e) Is this a power cycle or a refrigeration cycle?

Solution:

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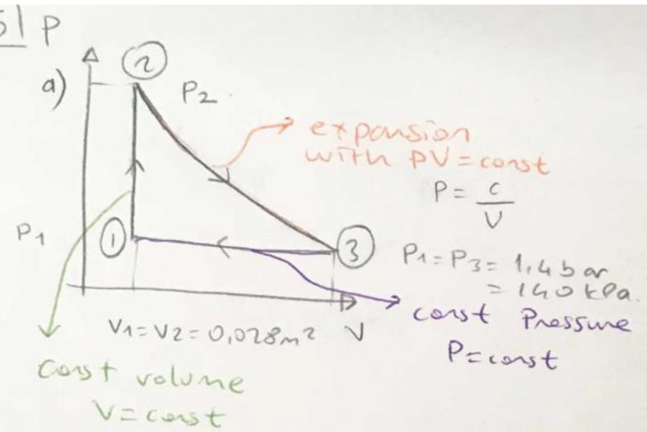
a)



1-2

$\Delta U = {}_1Q_2 - {}_1W_2$

$26,4 = {}_1Q_2 - {}_1W_2$

${}_1W_2 = \int_{V_1}^{V_2} P\,dV = 0 \quad (dV = 0)$

$\underline{{}_1Q_2 = 26,4\ kJ}$

$U_3 = U_2 \Rightarrow U_3 - U_1 = 26,4\ kJ$

2-3

$\Delta U = {}_2Q_3 - {}_2W_3$

$0 = {}_2Q_3 - {}_2W_3$

$\underline{{}_2W_3 = {}_2Q_3}$

${}_2W_3 = \int_{V_2}^{V_3} P\,dV = \int_{V_2}^{V_3} \frac{C}{V}\,dV$

3-1

$\Delta U = {}_3Q_1 - {}_3W_1$

$U_1 - U_3 = {}_3Q_1 - {}_3W_1$

$-26,4 = {}_3Q_1 - (-10,5)$

$\Rightarrow \boxed{{}_3Q_1 = -36,9\ kJ}$ d

${}_3W_1 = \int_{V_3}^{V_1} P\,dV = P_3 (V_1 - V_3)$

${}_3W_1 = 140\,(0,028 - V_3) = -10,5 \Rightarrow$ (work is done on the system)

$(0,028 - V_3) = \frac{-10,5}{140} = -0,075$

$\Rightarrow -V_3 = -0,075 - 0,028 = -0,103$

$\Rightarrow \underline{V_3 = 0,103\ m^3}$

2-3

So ${}_2W_3 = \int_{V_2}^{V_3} C \cdot \frac{1}{V}\,dV = C \cdot \ln V \Big|_{0,028}^{0,103} \quad 14,42 \cdot \ln \frac{0,103}{0,028}$

find $C \Rightarrow P_3 V_3 = C \Rightarrow 140 \cdot 0,103 = C = 14,42$

So ${}_2W_3 = 14,42 \cdot \ln \frac{0,103}{0,028} = 14,42 \cdot \ln 3,67 = \underline{18,78\ kJ}$

(work is done by the system)

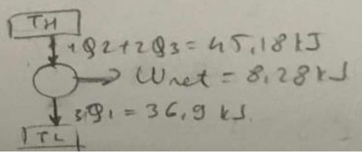
So ${}_2W_3 = {}_2Q_3 \Rightarrow \boxed{{}_2Q_3 = 18,78\ kJ}$ c

b) $W_{net} = {}_1W_2 + {}_2W_3 + {}_3W_1 = 0 + 18,78 - 10,5 = 8,28\ kJ$

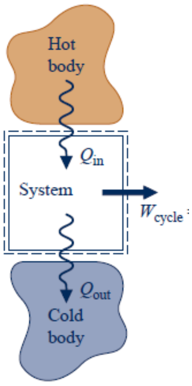
$\Rightarrow \boxed{W_{net} = 8,28\ kJ}$

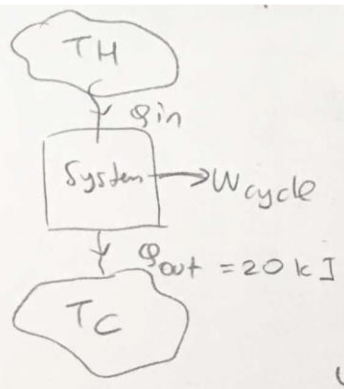
It is a POWER CYCLE!

$W_{net} > 0$



6) A power cycle shown Fig. has the thermal efficiency of **30%**, and $\mathbf{Q_{out}}$ is **20 kJ**. Determine the net work developed and the heat transfer $\mathbf{Q_{in}}$ in kJ.





$$\eta_{th} = \frac{W_{cycle}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$\eta_{th} = 0,3 = 1 - \frac{20}{Q_{in}} \Rightarrow \frac{20}{Q_{in}} = 0,7$$

$$Q_{in} = \frac{20}{0,7} = \underline{\underline{28,57 \text{ kJ}}}$$

$$W_{cycle} = Q_{in} - Q_{out}$$

$$W_{cycle} = 28,57 - 20 = 8,571$$

$$\rightarrow \underline{\underline{W_{cycle} = 8,57 \text{ kJ}}}$$